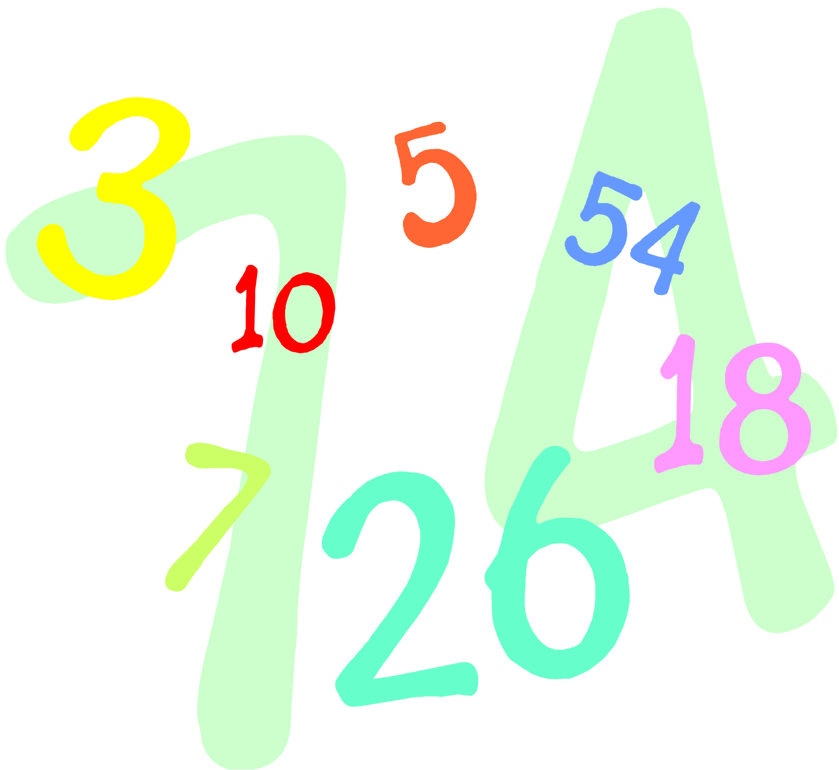
Lenzie Academy



Numeracy Booklet

A Guide

Introduction

What is Numeracy?
Numeracy is a skill for life, learning and work. Having well-developed numeracy skills allows young people to be more confident in social settings and enhances enjoyment in a large number of leisure activities.
Curriculum for Excellence

What is the purpose of the booklet?
This booklet has been produced to give guidance to staff & parents/carers on how certain common Numeracy topics are taught within the Mathematics department for problem solving, following the Curriculum for Excellence guidelines used in all schools in Scotland.

Curriculum for Excellence Numeracy Strands

- Estimation and Rounding
- Number and number processes
- Fractions, Decimals and Percentages
- Money
- Time
- Measurement
- Data and Analysis
- Ideas of chance and uncertainty

How can it be used?
Before teaching a topic containing numeracy you can refer to the booklet to see what methods are being taught.

Why do some topics include more than one method?
In some cases (e.g. percentages), the method used will be dependent on the level of difficulty of the question, and whether or not a calculator is permitted.

For mental calculations, pupils should be encouraged to develop a variety of strategies so that they can select the most appropriate method in any given situation.

For calculator questions do try to estimate the answer mentally first.

Mathematics Department, Lenzie Academy 2017.

The following guide gives an indication as to when a pupil could expect to see a particular numeracy topic in their Primary or Secondary Mathematics class.

work with numbers	**Level**
up to 20	1
up to 1000	1
up to 10 000	2
up to 1 000 000	2
multiples and factors of numbers	2
negative numbers	2
orders of operations	2
common multiples and factors	3
prime numbers	3
powers and roots	3
solve problems using negative numbers in context	3
add, subtract, multiply and divide negative numbers	4
indices and scientific notation	4
tolerance in number	4
calculate without a calculator	
add / subtract 2 digit numbers	1
add / subtract 2 digit and 3 digit numbers	1
multiply and divide 2 digits by 2, 3, 4, 5, 10	1
add / subtract 4 digit numbers including decimals	2
multiply and divide 2 digit by single digit	2
multiply and divide 4 digit by single digit	2
multiply and divide any number of digits up to 3 dp	3
multiply and divide any number of digits	3
multiply and divide decimals by decimals	4
rounding numbers	
2 digit to nearest 10	1
3 digit to nearest 10	1
any number to nearest 10, 100	2
any number to 1 decimal place	3
as required including significant figures	3
work with fractions and percentages	
halves, quarters of quantities	1
thirds, fifths, tenths of quantities	1
simple fraction of a quantity	1
equivalence of widely used fractions and percentages	1
widely used fractions of whole numbers	2
identify a simple ratio	2
equivalence of fractions, ratios, percentages	2
add, subtract, multiply and divide fractions	3
mixed numbers and improper fractions	3

work with fractions and percentages (continued)	**Level**
unitary ratio	3
sharing from a ratio	3
direct proportion	3
percentage of whole number	3
percentage increase and decrease	4
reversing the percentage change	4
operations with fractions and mixed numbers	4
work with time	
days, seasons, tell time in hours	early
read digital and half and quarter on analogue clock	1
12 hour clock, time intervals less than 1 hour, calendar	1
24 hour clock, time interval in hours and minutes	2
tenths, hundredths of seconds from stopwatches	3
speed, distance, time calculations	3
length	
handspans, non-standard units	early
metre and centimetre	1
millimetre, kilometre, common imperial	2
perimeter	2
weight	
non-standard units	early
kilogram, gram, estimating weight	1
use and convert between units	2
capacity	
non-standard units	early
litres	1
volume	1
millilitres	2
converting units of capacity $1cm^3 = 1ml$, 1 litre = 1000ml	3
area	
non-standard units	early
find area using squared paper	1
area of a triangle using squared paper	1
cm^2, m^2, km^2, hectares	2
area of a square and rectangle using formula	3
area of a kite, rhombus, parallelogram	3
area of a circle	3
circumference of a circle	3
converting units of area, m^2 to km^2, m^2 to hectares	3
surface area of 3D objects	4

volume	**Level**
rules for cube and cuboid	3
converting units of volume, $1m^3 = 1000000cm^3$	3
volume of prisms	4
temperature	
above zero	2
below zero	2
measuring	
to nearest labelled graduation	1
to nearest graduation	1
by estimating between graduations	2
collection by survey	
1 direct question	1
yes / no questionnaire	1
questionnaire with several responses	2
simple sampling strategy	3
structured questionnaire multi-response	4
sampling avoiding bias	4
organising information	
tally without grouping	1
tally in groups	2
use tables to record	2
design and use tables (frequency tables)	3
grouping discrete / continuous data	4
cumulative frequencies	4
displaying information	
bar graph with unit scale	1
bar graph with scale in multiples	1
bar graph and pie chart (simple fractions)	2
line graph and frequency polygon	2
using spreadsheets	2
extended use of pie charts inc. from raw data	3
construction and analysis of extended range of displays	3
curved graphs	3
scatter graphs and stem and leaf	4
interpreting information	
answer direct question	1
identify most and least	1
retrieve information subject to 1 condition	1
retrieve information subject to more than one condition	2
describe features	2
retrieve information from range of displays	2

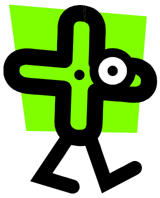
interpreting information (continued)	Level
retrieve information from extended range of displays	3
comparing data sets (e.g. comparative line graphs)	3
distribution and trends	3
misleading data	3
describe correlation	4
mean, median, mode, range	4
discrete and continuous data	4

chance and uncertainty	
simple probability, certain events, impossible events	2
use of ‘impossible’, ‘unlikely’, ‘evens’, ‘very likely’, ‘certain’	3
using formula to define the probability of an event	4

Table of Contents

Addition

Mental strategies



There are a number of useful mental strategies for addition. Some examples are given below.

Example Calculate 64 + 27

Method 1 Add tens, then add units, then add together

60 + 20 = 80   4 + 7 = 11   80 + 11 = 91

Method 2 Split up number to be added (last number 27) into tens and units and add separately.

64 + 20 = 84   84 + 7 = 91

Method 3 Round up to nearest 10, then subtract

64 + 30 = 94 but 30 is 3 too much so subtract 3;
94 - 3 = 91

Written Method

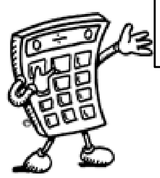
When adding numbers, ensure that the numbers are lined up according to place value. Start at right hand side, write down units, carry tens.

Example Add 3032 and 589

$$\begin{array}{r} 3032 \\ +589 \\ \hline {\scriptstyle 1} \\ 1 \\ \hline \end{array} \rightarrow \begin{array}{r} 3032 \\ +589 \\ \hline {\scriptstyle 1\,1} \\ 21 \\ \hline \end{array} \rightarrow \begin{array}{r} 3032 \\ +589 \\ \hline {\scriptstyle 1\,1} \\ 621 \\ \hline \end{array} \rightarrow \begin{array}{r} 3032 \\ +589 \\ \hline {\scriptstyle 1\,1} \\ 3621 \\ \hline \end{array}$$

2 + 9 = 11   3+8+1=12   0+5+1=6   3 + 0 = 3

Subtraction



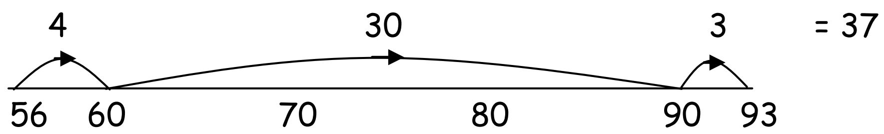
We use decomposition as a written method for subtraction (see below). Alternative methods may be used for mental calculations.

Mental Strategies

Example Calculate 93 - 56

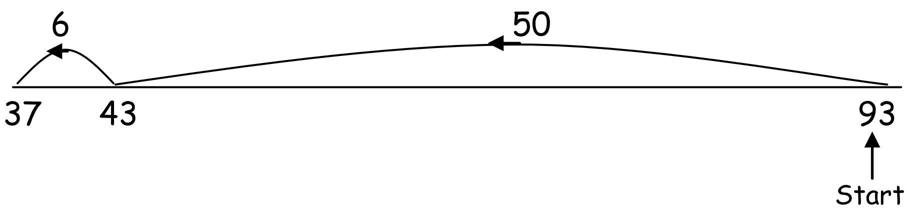
Method 1 Count on

Count on from 56 until you reach 93. This can be done in several ways e.g.



Method 2 Break up the number being subtracted

e.g. subtract 50, then subtract 6 93 - 50 = 43
43 - 6 = 37



Written Method

Example 1 4590 – 386

$$\begin{array}{r} 45\overset{8}{\cancel{9}}{}^{1}0 \\ -\ 386 \\ \hline 4204 \\ \hline \end{array}$$

We do not "borrow and pay back".

Example 2 Subtract 692 from 14597

$$\begin{array}{r} 1\overset{3}{\cancel{4}}{}^{1}597 \\ -\ \ 692 \\ \hline 13905 \\ \hline \end{array}$$

Important steps for example 1

1. Say "zero take away 6, we can't do"
2. Look to next column exchange one ten for ten units ie 9tens becomes 8tens & 10units
3. Then say "ten take away six equals four"
4. Normal subraction rules can be used to then complete the question.

Multiplication 1

It is essential that you know all of the multiplication tables from 1 to 12. These are shown in the tables square below.

×	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Mental Strategies

Example Find 39 x 6

Method 1

30 x 6
= 180

9 x 6
= 54

180 + 54
= 234

Method 2

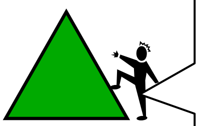
40 x 6
=240

40 is 1 too many
so take away 1x6

240 - 6
= 234

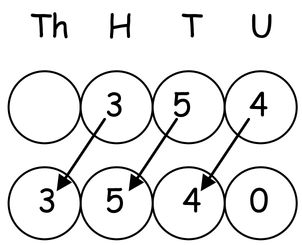
Multiplication 2

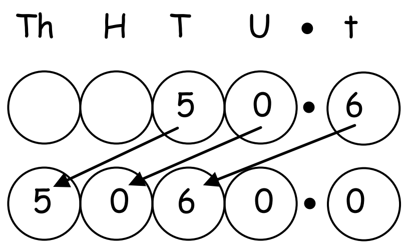
Multiplying by multiples of 10 and 100



To multiply by **10** you move every digit ***one*** place to the left (or add one zero to a whole number).
To multiply by **100** you move every digit ***two*** places to the left (or add 2 zeroes to a whole number).

Example 1 (a) Multiply 354 by 10 (b) Multiply 50.6 by 100





354 x 10 = 3540

50.6 x 100 = 5060

(c) 35 x 30

To multiply by 30, multiply by 3, then by 10.

35 x 3 = 105
105 x 10 = 1050

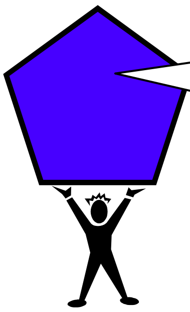
so 35 x 30 = 1050

(d) 436 x 600

To multiply by 600, multiply by 6, then by 100.

436 x 6 = 2616
2616 x 100 = 261600

so 436 x 600 = 261600



We may also use these rules for multiplying decimal numbers.

Example 2 (a) 2.36 x 20

2.36 x 2 = 4.72
4.72 x 10 = 47.2

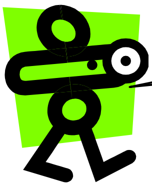
so 2.36 x 20 = 47.2

(b) 38.4 x 50

38.4 x 5 = 192.0
192.0x 10 = 1920

so 38.4 x 50 = 1920

Division



You should be able to divide by a single digit or by a multiple of 10 or 100 without a calculator.

Written Method

Example 1 There are 192 pupils in first year, shared equally between 8 classes. How many pupils are in each class?

$$\begin{array}{r|l} & \ \ 2\ 4 \\ \hline 8 & 1\ 9\ ^{3}2 \end{array}$$

There are 24 pupils in each class

Example 2 Divide 4.74 by 3

$$\begin{array}{r|l} & 1\ .\ 5\ 8 \\ \hline 3 & 4\ .\ ^{1}7\ ^{2}4 \end{array}$$

When dividing a decimal number by a whole number, the decimal points must stay in line.

Example 3 A jug contains 2.2 litres of juice. If it is poured evenly into 8 glasses, how much juice is in each glass?

$$\begin{array}{r|l} & 0\ .\ 2\ 7\ 5 \\ \hline 8 & 2\ .\ ^{2}2\ ^{6}0\ ^{4}0 \end{array}$$

Each glass contains 0.275 litres

If you have a remainder at the end of a calculation, add a zero onto the end of the decimal and continue with the calculation.

Long Division – This was not in the 5-14 course nor is it part of the Numeracy outcomes. Pupils would estimate the answer and then use a calculator to get the exact answer.

Estimation and Rounding

Estimating

As a guide pupils should be able to:

estimate height and length in centimetres and metres
e.g. length of pencil = 10cm, width of classroom = 7m

estimate small weights, small areas, small volumes
e.g. bag of sugar = 1kg, area of text book page = 200 cm^2,
volume of a mug = 300 ml

estimate areas in square metres, lengths in mm
e.g. area of the room = $70m^2$, diameter of 1p = 15mm

estimate the answer to a calculation without a calculator
e.g £599 ÷ 18

£600 ÷ 20 = £30

Round each number first

Rounding

We only consider the first digit after the required accuracy digit for rounding purposes

Rules for rounding:
4 or below, round down
5 or above, round up

Examples,

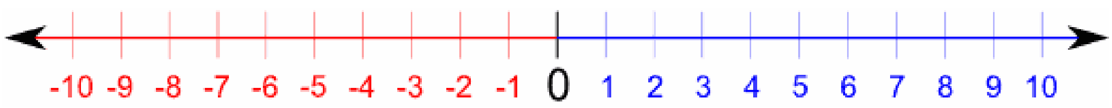
63 rounded to the nearest 10 is 60
749 rounded to the nearest 10 is 750

234.7 rounded to the nearest whole number is 235
234.7 rounded to the nearest 10 is 230
234.7 rounded to the nearest 100 is 200

9.76 rounded to 1 decimal place is 9.8
2.3456 rounded to 2 decimal places is 2.35
2.3456 rounded to 2 significant figures is 2.3

Integers

The set of numbers known as integers comprises positive and negative whole numbers and the number zero. Negative numbers are below zero and are written with a negative sign, " – ". Integers can be represented on a number line.



Negative Numbers (-) Positive Numbers (+)

(The line continues left and right forever.)

Integers are used in a number of real life situations including profit and loss, temperature, height below sea level and golf scores.

Adding and Subtracting Integers

Consider 2 + 3. Using a number line this addition would be "start at 2 and move right 3 places". Whereas 2 – 3 would be "start at 2 and move left 3 places". Picturing a number line may help pupils extend their addition and subtraction to integers.

Examples

1. -4 + 3 = -1	start at -4 and move 3 places to the right	2. -8 + 10 = 2	start at -8 and move 10 places to the right
3. -5 – 2 = -7	start at -5 and move 2 places to the left	4. -11 – 7 = -18	start at -11 and move 7 places to the left

Now consider 2 + (-3). Here we "start at 2 and move 3 places to the left because of the (-3). Therefore, 2 + (-3) = -1 (the same as 2 – 3 = -1).

Similarly, 2 – (-3) means to start at 2 and move 3 places to the right. So 2 – (-3) = 5

Examples

1. 4 + (-6)	2. -7 + (-8)	3. -11 + (-5)	4. 6 – (-4)	5. -3 – (-5)	6. -8 – (-2)
= 4 – 6	= -7 – 8	= -11 – 5	= 6 + 4	= -3 + 5	= -8 + 2
= -2	= -15	= -16	= 10	= 2	= -6

7. What is the difference in temperature between -14°C and -51°C?
-14 – (-51)
= -14 + 51
= 37°C

Order of Calculation (BODMAS)

Consider this: What is the answer to 2 + 4 x 5 ?

Is it	(2+4) x 5	or	2 + (4 x 5)
	= 6 x 5		= 2 + 20
	= 30		= 22

The correct answer is 22.

Calculations which have more than one operation need to be done in a particular order. The order can be remembered by using the mnemonic **BODMAS**

The **BODMAS** rule tells us which operations should be done first.
BODMAS represents:

(B)rackets
(O)rder
(D)ivide
(M)ultiply
(A)dd
(S)ubract

Therefore in the example above multiplication should be done before addition. (Note order means a number raised to a power such as 2^2 or $(-3)^3$)

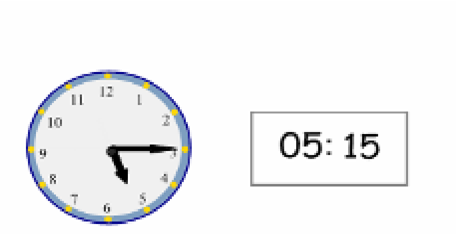
Scientific calculators are programmed with these rules, however some basic calculators may not so take care.

Example 1	15 – 12 ÷ 6	BODMAS tells us to divide first
	= 15 – 2	
	= 13	
Example 2	(9 + 5) x 6	BODMAS tells us to work out the brackets first
	= 14 x 6	
	= 84	
Example 3	18 + 6 ÷ (5-2)	Brackets first
	= 18 + 6 ÷ 3	Then divide
	= 18 + 2	Now add
	= 20	
Example 4	4 x 5^2	The power takes priority
	= 4 x 25	Then multiply
	= 100	

Time 1

Time may be expressed in 12 or 24 hour notation.

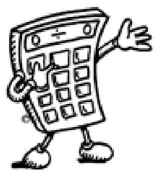
12-hour clock Time can be displayed on a clock face, or digital clock.



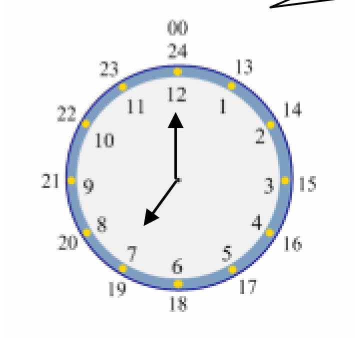
These clocks both show fifteen minutes past five, or quarter past five.

When writing times in 12 hour clock, we need to add a.m. or p.m. after the time.
a.m. is used for times between midnight and 12 noon (morning)
p.m. is used for times between 12 noon and midnight (afternoon / evening).

24-hour clock



In 24 hour clock, the hours are written as numbers between 00 and 24. Midnight is expressed as 00:00, or 24:00. After 12 noon, the numbers are numbered 13, 14, 15... etc.



Examples

9.55 am	→	09:55 hours
3.35 pm	→	15:35 hours
12.20 am	→	00:20 hours
02:16 hours	→	2.16 am
20:45 hours	→	8.45 pm
12:30 hours	→	12.30 pm

Reading timetables

When reading timetables you often have to convert to and from 24 hour clock.

To convert from 24 hour time to 12 hour time:

A. If the hour is 13 or more, subtract 12 from the hours and call it p.m. Otherwise it is a.m.
B. If the hour is 12, leave it unchanged, but call it p.m.
C. If the hour is 00, make it 12 and call it a.m.
D. Otherwise, leave the hour unchanged and call it a.m.

To convert from 12-hour time to 24-hour time:

A. If the p.m. hour is from 1 through to 11, add 12.
B. If the p.m. hour is 12, leave it as is.
C. If the a.m. hour is 12, make it 00.
D. Otherwise, leave the hour unchanged.
Then drop the a.m. or p.m., of course.

***************Check rules with examples above***********************

Time 2

Time Facts

It is essential to know the number of months, weeks and days in a year, and the number of days in each month.

Time Calculations

Example 1 How long is it from 0755 to 0948?

Method - Working (use a time line)

0755	->	0800	->	9000	->	0948
	(5mins)		+ (1hr)		+ (48mins)	

WE DON'T TEACH TIME AS SUBTRACTION

Example 2 Change 27 minutes into hours equivalent

27mins = 27÷60

= 0.45 hours

Example 3 Change 2.41667 hours into hours and minutes

The whole numbers represent the amount of complete hours.

Here, it is 2 hours.

To work out how many minutes there are, subtract the whole numbers and multiply the remaining decimal by 60.

0.41667 x 60 = 25 minutes

So, 2.41667 = 2 hours and 25 minutes

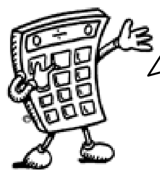
Common decimal time conversions (these should be learned by heart)

0.25 = 15 mins

0.5 = 30 mins

0.75 = 45 mins

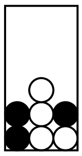
Fractions 1



Addition, subtraction, multiplication and division of fractions are studied in mathematics.
However, the examples below may be helpful in all subjects.

Understanding Fractions

Example
A jar contains black and white sweets.



What fraction of the sweets are black?

There are 3 black sweets out of a total of 7, so $\frac{3}{7}$ of the sweets are black.

Equivalent Fractions

Example
What fraction of the flag is shaded?

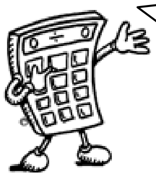
6 out of 12 squares are shaded. So $\frac{6}{12}$ of the flag is shaded.

It could also be said that $\frac{1}{2}$ the flag is shaded.

$\frac{6}{12}$ and $\frac{1}{2}$ are **equivalent fractions**.

Fractions 2

Simplifying Fractions



The top of a fraction is called the **numerator**, the bottom is called the **denominator**.
To simplify a fraction, divide the **numerator** and **denominator** of the fraction by the same number.

Example 1

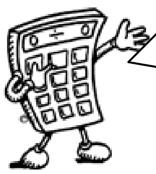
(a) $\frac{20}{25} \overset{\div 5}{\underset{\div 5}{=}} \frac{4}{5}$

(b) $\frac{16}{24} \overset{\div 8}{\underset{\div 8}{=}} \frac{2}{3}$

This can be done repeatedly until the numerator and denominator are the smallest possible numbers - the fraction is then said to be in it's **simplest form**.

Example 2 Simplify $\frac{72}{84}$ $\frac{72}{84} = \frac{36}{42} = \frac{18}{21} = \frac{6}{7}$ (simplest form)

Calculating Fractions of a Quantity



To find the fraction of a quantity, divide by the denominator.
To find $\frac{1}{2}$ divide by 2, to find $\frac{1}{3}$ divide by 3, to find $\frac{1}{7}$ divide by 7 etc.

Example 1 Find $\frac{1}{5}$ of £150

$\frac{1}{5}$ of £150 = £150 ÷ 5 = £30

Example 2 Find $\frac{3}{4}$ of 48

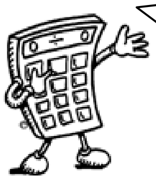
$\frac{1}{4}$ of 48 = 48 ÷ 4 = 12

so $\frac{3}{4}$ of 48 = 3 x 12 = 36

To find $\frac{3}{4}$ of a quantity, start by finding $\frac{1}{4}$ then multiply by 3 (the numerator)

Fractions 3

Adding, Subtracting Fractions



To add and subtract fractions you have to make the denominators the same by using equivalent fractions. Then you add or subtract the new numerators.

Example 1

$$\frac{1}{5}+\frac{3}{5}=\frac{4}{5}$$

both fractions have the same denominator so we can simply add the numerators

Example 2

$$\frac{2}{3}-\frac{1}{4}$$

$$\frac{8}{12}-\frac{3}{12}$$

$$\frac{5}{12}$$

Make denominators the same. Then subtract new numerators.

Multiplying, Dividing Fractions

To multiply fractions, multiply the numerators together and the denominators together, separately. To divide, invert the second fraction (turn upside down) and then multiply.

Example 1

$$\frac{4}{5}\times\frac{2}{3}=\frac{8}{15}$$

Example 2

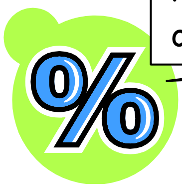
$$\frac{2}{7}\div\frac{1}{3}$$

$$\frac{2}{7}\times\frac{3}{1}$$

remember to invert second fraction

$$\frac{6}{7}$$

Percentages 1



Percent means out of 100.
A percentage can be converted to an equivalent fraction or decimal.

36% means $\frac{36}{100}$

36% is therefore equivalent to $\frac{9}{25}$ and 0.36

To change a fraction to a decimal (fraction) divide the numerator by the denominator

Common Percentages

Some percentages are used very frequently. It is useful to know these as fractions and decimals.

Percentage	Fraction	Decimal (Fraction)
1%	$\frac{1}{100}$	0.01
10%	$\frac{1}{10}$	0.1
20%	$\frac{1}{5}$	0.2
25%	$\frac{1}{4}$	0.25
$33^{1}/_{3}\%$	$\frac{1}{3}$	0.333…
50%	$\frac{1}{2}$	0.5
$66^{2}/_{3}\%$	$\frac{2}{3}$	0.666…
75%	$\frac{3}{4}$	0.75

Percentages 2

There are many ways to calculate percentages of a quantity. Some of the common ways are shown below.

Non- Calculator Methods

Method 1 Using Equivalent Fractions

Example Find 25% of £640

25% of £640 $= \frac{1}{4}$ of £640 = £640 ÷ 4 = £160

Method 2 Using 1%

In this method, first find 1% of the quantity (by dividing by 100), then multiply to give the required value.

Example Find 9% of 200g

1% of 200g = $\frac{1}{100}$ of 200g = 200g ÷ 100 = 2g

so 9% of 200g = 9 x 2g = 18g

Method 3 Using 10%

This method is similar to the one above. First find 10% (by dividing by 10), then multiply to give the required value.

Example Find 70% of £35

10% of £35 = $\frac{1}{10}$ of £35 = £35 ÷ 10 = £3.50

so 70% of £35 = 7 x £3.50 = £24.50

Percentages 3

Non- Calculator Methods (continued)

The previous 2 methods can be combined so as to calculate any percentage.

Example Find 23% of £15000

10% of £15000 = £1500 so 20% = £1500 x 2 = £3000
1% of £15000 = £150 so 3% = £150 x 3 = £450

23% of £15000 = £3000 + £450 = £3450

Finding VAT (without a calculator)

Value Added Tax (VAT) = 17.5%
To find VAT, firstly find 10%

Example Calculate the total price of a computer which costs £650 excluding VAT

10% of £650 = £65 (divide by 10)
5% of £650 = £32.50 (divide previous answer by 2)
2.5% of £650 = £16.25 (divide previous answer by 2)

so 17.5% of £650 = £65 + £32.50 + £16.25 = £113.75

Total price = £650 + £113.75 = £768.75

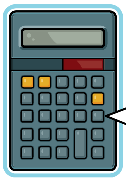
Percentages 4

Calculator Method

To find the percentage of a quantity using a calculator, change the percentage to a decimal, then multiply.

Example 1 Find 23% of £15000

23% = 0.23 so 23% of £15000 = 0.23 x £15000 = £3450



We do **not** use the % button on calculators. The methods taught in the mathematics department are all based on converting percentages to decimals.

Example 2 House prices increased by 19% over a one year period. What is the new value of a house which was valued at £236000 at the start of the year?

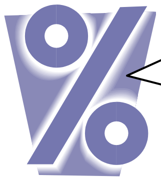
19% = 0.19 so Increase = 0.19 x £236000
= £44840

Value at end of year = original value + increase
= £236000 + £44840
= £280840

The new value of the house is £280840

Percentages 5

Finding the percentage



To find a percentage of a total, first make a fraction, then convert to a decimal by dividing the top by the bottom. This can then be expressed as a percentage.

Example 1 There are 30 pupils in Class 3A3. 18 are girls. What percentage of Class 3A3 are girls?

$$\frac{18}{30} = 18 \div 30 = 0.6 = 60\%$$

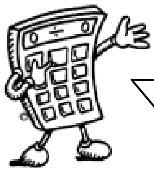
60% of 3A3 are girls

Example 2 James scored 36 out of 44 in his biology test. What is his percentage mark?

$$\text{Score} = \frac{36}{44} = 36 \div 44 = 0.81818\ldots$$

$$= 81.818..\% = 82\% \text{ (rounded)}$$

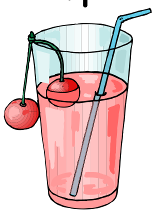
Ratio 1



When quantities are to be mixed together, the ratio, or proportion of each quantity is often given. The ratio can be used to calculate the amount of each quantity, or to share a total into parts.

Writing Ratios

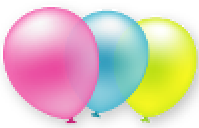
Example 1



To make a fruit drink, 4 parts water is mixed with 1 part of cordial.
The ratio of water to cordial is 4:1
(said "4 to 1")
The ratio of cordial to water is 1:4.

Order is important when writing ratios.

Example 2



In a bag of balloons, there are 5 red, 7 blue and 8 green balloons.

The ratio of red : blue : green is 5 : 7 : 8

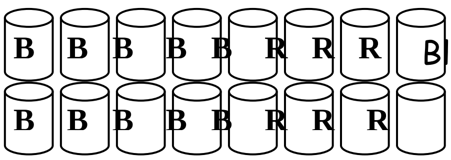
Simplifying Ratios

Ratios can be simplified in much the same way as fractions.

Example 1

Purple paint can be made by mixing 10 tins of blue paint with 6 tins of red. The ratio of blue to red can be written as 10 : 6

It can also be written as 5 : 3, as it is possible to split up the tins into 2 groups, each containing 5 tins of blue and 3 tins of red.



Blue : Red = 10 : 6
= 5 : 3

To simplify a ratio, divide each figure in the ratio by a common factor.

Ratio 2

Simplifying Ratios (continued)

Example 2

Simplify each ratio:

(a) 4:6 (b) 24:36 (c) 6:3:12

(a) 4:6
= 2:3

Divide each figure by 2

(b) 24:36
= 2:3

Divide each figure by 12

(c) 6:3:12
= 2:1:4

Divide each figure by 3

Example 3

Concrete is made by mixing 20 kg of sand with 4 kg cement. Write the ratio of sand : cement in its simplest form

Sand : Cement = 20 : 4
= 5 : 1

Using ratios

The ratio of fruit to nuts in a chocolate bar is 3 : 2. If a bar contains 15g of fruit, what weight of nuts will it contain?

Fruit	Nuts
3	2
x5	x5
15	**10**

So the chocolate bar will contain 10g of nuts.

Information Handling : Tables

It is sometimes useful to display information in graphs, charts or tables.

Example 1 The table below shows the average maximum temperatures (in degrees Celsius) in Barcelona and Edinburgh.

	J	F	M	A	M	J	J	A	S	O	N	D
Barcelona	13	14	15	17	20	**24**	27	27	25	21	16	14
Edinburgh	6	6	8	11	14	17	18	18	16	13	8	6

The average temperature in June in Barcelona is 24°C

Frequency Tables are used to present information. Often data is grouped in intervals.

Example 2 Homework marks for Class 4B

27 30 23 24 22 35 24 33 38 43 18 29 28 28 27
33 36 30 43 50 30 25 26 37 35 20 22 24 31 48

Mark	Tally	Frequency
16 - 20	||	2
21 - 25	~~||||~~ ||	7
26 - 30	~~||||~~ ||||	9
31 - 35	~~||||~~	5
36 - 40	|||	3
41 - 45	||	2
46 - 50	||	2

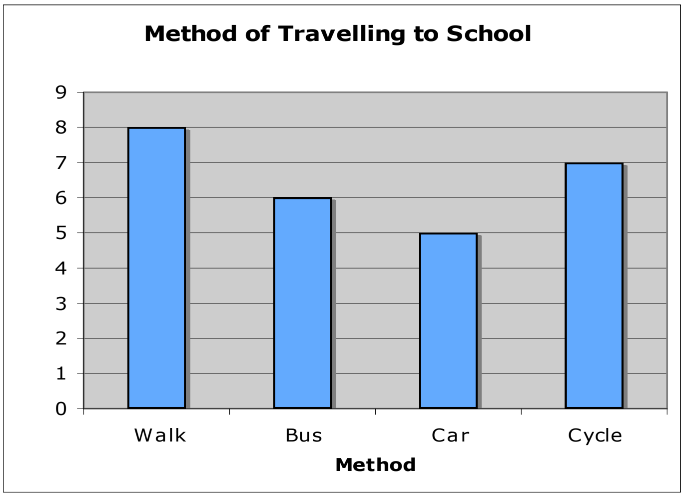
Each mark is recorded in the table by a tally mark.
Tally marks are grouped in 5's to make them easier to read and count.

Information Handling : Bar Graphs/Histograms

Bar graphs and Histograms are often used to display data. <u>They must not be confused as being the same</u>.
<u>Bar graphs</u> are used to present discrete* or non numerical data* whereas <u>histograms</u> are used to present continuous data*.
See key words (Page 34) for explanation of these terms
All graphs should have a title, and each axis must be labelled.

Example 1 Example of a Bar Graph

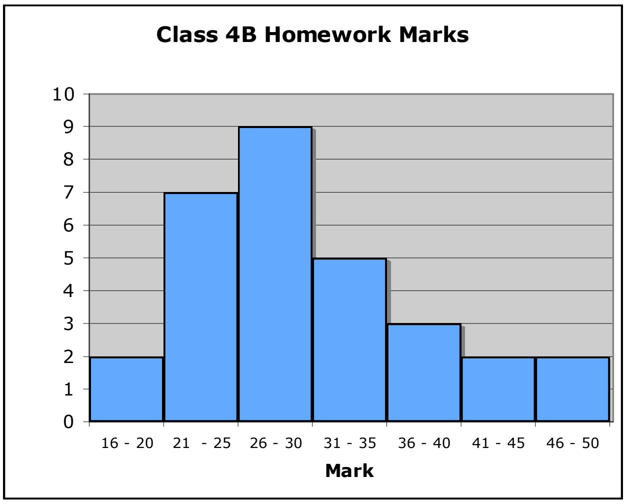
How do pupils travel to school?



An even space should be between each bar and each bar should be of an equal width. (also leave a space between vertical axis and the first bar.)

Example 2 Example of a histogram

The graph below shows the homework marks for Class 4B.

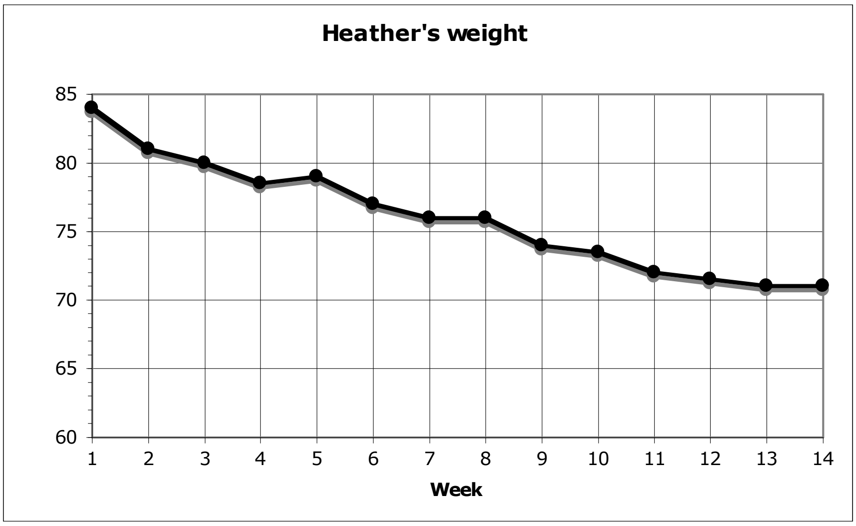


Important – there should be no space between each bar

Information Handling : Line Graphs

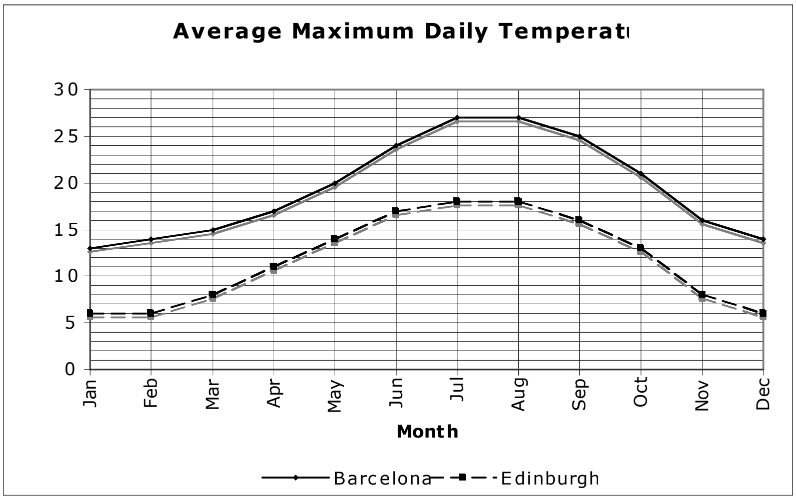
Line graphs consist of a series of points which are plotted, then joined by a line. All graphs should have a title, and each axis must be labelled. The trend of a graph is a general description of it.

Example 1 The graph below shows Heather's weight over 14 weeks as she follows an exercise programme.

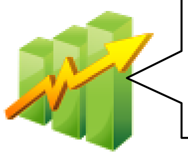


The trend of the graph is that her weight is decreasing.

Example 2 Graph of temperatures in Edinburgh and Barcelona.



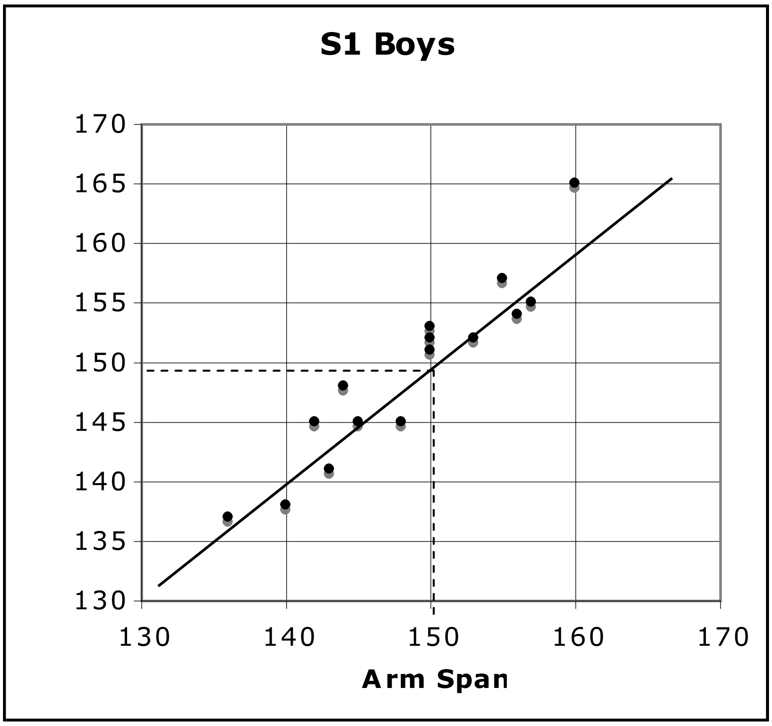
Information Handling : Scatter Graphs



A scatter diagram is used to display the relationship between two variables.
A pattern may appear on the graph. This is called a **correlation**.

Example The table below shows the height and arm span of a group of first year boys. This is then plotted as a series of points on the graph below.

Arm Span (cm)	150	157	155	142	153	143	140	145	144	150	148	160	150	156	136
Height (cm)	153	155	157	145	152	141	138	145	148	151	145	165	152	154	137

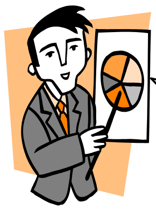


The graph shows a general trend, that as the arm span increases, so does the height. This graph shows a positive correlation.

The line drawn is called the line of best fit. This line can be used to provide estimates. For example, a boy of arm span 150cm would be expected to have a height of around 149cm.

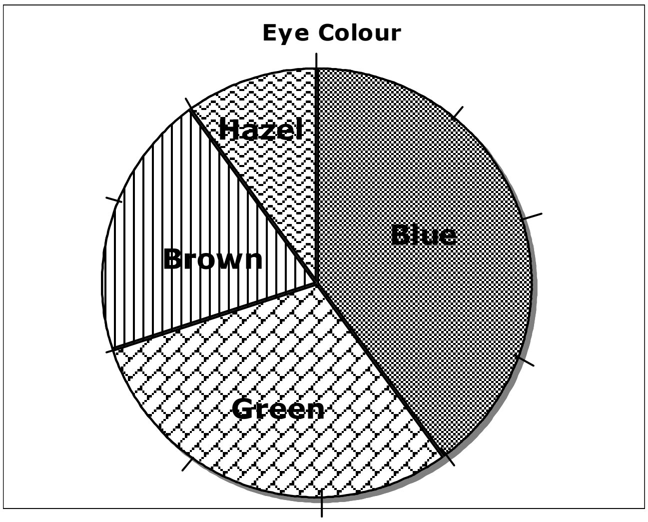
Note that in some subjects, it is a requirement that the axes start from zero.

Information Handling : Pie Charts



A pie chart can be used to display information. Each sector (slice) of the chart represents a different category. The size of each category can be worked out as a fraction of the total using the number of divisions or by measuring angles.

Example 30 pupils were asked the colour of their eyes. The results are shown in the pie chart below.



How many pupils had brown eyes?

The pie chart is divided up into ten parts, so pupils with brown eyes represent $\frac{2}{10}$ of the total.

$\frac{2}{10}$ of 30 = 6 so 6 pupils had brown eyes.

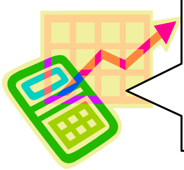
If no divisions are marked, we can work out the fraction by measuring the angle of each sector.

The angle in the brown sector is 72°.
so the number of pupils with brown eyes

$= \frac{72}{360} \times 30 = 6$ pupils.

If finding all of the values, you can check your answers - the total should be 30 pupils.

Information Handling : Averages



To provide information about a set of data, the average value may be given. There are 3 ways of finding the average value – the mean, the median and the mode.

Mean

The mean is found by adding all the data together and dividing by the number of values.

Median

The median is the middle value when all the data is written in numerical order (if there are two middle values, the median is half-way between these values).

Mode

The mode is the value that occurs most often.

Range

The range of a set of data is a measure of spread.

Range = Highest value – Lowest value

Example Class 1A scored the following marks for their homework assignment. Find the mean, median, mode and range of the results.

6, 9, 7, 5, 6, 6, 10, 9, 8, 4, 8, 5, 7

$$\text{Mean} = \frac{6+9+7+5+6+6+10+9+8+4+8+5+7}{13}$$

$$= \frac{90}{13} = 6.923...$$

Mean = 6.9 to 1 decimal place

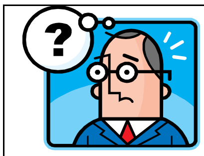
Ordered values: 4, 5, 5, 6, 6, 6, $\boxed{7}$ 7, 8, 8, 9, 9, 10

Median = 7

6 is the most frequent mark, so Mode = 6

Range = 10 – 4 = 6

Chance and Uncertainty



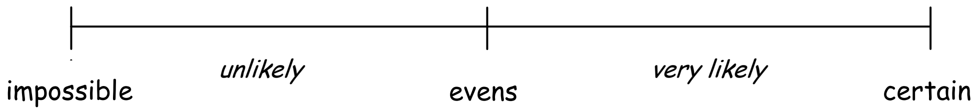
Data is often collected in order to find out the chance that a certain event will occur.

The **probability** or chance that something happens is measured on a Scale from 0 to 1.

It can also be described using words, such as likely or unlikely, but these words can often be vague.

If an event is **certain** to happen, it has a probability of **1**.

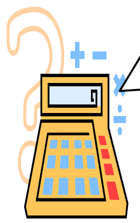
If an event is **impossible**, it has a probability of **0**.



It is possible to calculate the probability of an event using the formula,

$$\text{Probability of an event} = \frac{\text{number of ways that event can occur}}{\text{total number of different outcomes}}$$

Scientific Notation or Standard Form



In engineering and scientific calculations you often deal with very small or very large numbers, for example 0.00000345 and 870,000,000.
To avoid writing these very long numbers a system has been developed, known as **scientific notation (standard form)** which enables us to write numbers much more concisely.

The rules when writing a number in standard form is that first you write down a number between 1 and 10, then you write × 10 (to the power of a number).

Example

Write 81 900 000 000 000 in standard form:

81 900 000 000 000 = 8.19×10^{13}

It's 10^{13} because the decimal point has been moved 13 places to the left to get the number to be 8.19

Example

Write 0.000 001 2 in standard form:

0.000 001 2 = 1.2×10^{-6}

It's 10^{-6} because the decimal point has been moved 6 places to the right to get the number to be 1.2

On a calculator, you usually enter a number in standard form as follows:
Type in the first number (the one between 1 and 10). Press EXP . Type in the power to which the 10 is risen.

Interesting facts

Mass of Earth	**= 5974200000000000000000000 kg**
	= 5.9742×10^{24} kg
Mass of an electron	**=0.00000000000000000000000000000092**
	= 9.2×10^{-31} kg

Mathematical literacy (Key words):

Add; Addition (+)	To combine 2 or more numbers to get one number (called the sum or the total) Example: 12+76 = 88
a.m.	(ante meridiem) Any time in the morning (between midnight and 12 noon).
Approximate	An estimated answer, often obtained by rounding to nearest 10, 100 or decimal place.
Calculate	Find the answer to a problem. It doesn't mean that you must use a calculator!
Continuous Data	Has an infinite number of possible values within a selected range e.g. temperature, height,length
Data	A collection of information (may include facts, numbers or measurements).
Discrete	Can only have a finite or limited number of possible values. Shoe sizes are an example of discrete data because sizes 6 and 7 mean something, but size 6.3 for example does not.
Denominator	The bottom number in a fraction (the number of parts into which the whole is split).
Difference (-)	The amount between two numbers (subtraction). Example: The difference between 50 and 36 is 14 50 – 36 = 14
Division (÷)	Sharing a number into equal parts. 24 ÷ 6 = 4
Double	Multiply by 2.
Equals (=)	Makes or has the same amount as.
Equivalent fractions	Fractions which have the same value. Example $\frac{6}{12}$ and $\frac{1}{2}$ are equivalent fractions
Estimate	To make an approximate or rough answer, often by rounding.
Evaluate	To work out the answer.
Even	A number that is divisible by 2. Even numbers end with 0, 2, 4, 6 or 8.
Factor	A number which divides exactly into another number, leaving no remainder. Example: The factors of 15 are 1, 3, 5, 15.
Frequency	How often something happens. In a set of data, the number of times a number or category occurs.

Greater than (>)	Is bigger or more than. Example: 10 is greater than 6. 10 > 6
Least	The lowest number in a group (minimum).
Less than (<)	Is smaller or lower than. Example: 15 is less than 21. 15 < 21.
Maximum	The largest or highest number in a group.
Mean	The arithmetic average of a set of numbers - see p26
Median	Another type of average - the middle number of an ordered set of data - see p26
Minimum	The smallest or lowest number in a group.
Minus (-)	To subtract.
Mode	Another type of average – the most frequent number or category (see p26)
Most	The largest or highest number in a group (maximum).
Multiple	A number which can be divided by a particular number, leaving no remainder. Example Some of the multiples of 4 are 8, 16, 48, 72
Multiply (x)	To combine an amount a particular number of times. Example 6 x 4 = 24
Negative Number	A number less than zero. Shown by a minus sign. Example -5 is a negative number.
Numerator	The top number in a fraction.
Non Numerical data	Data which is non numerical e.g. favourite football team, favourite sweet etc.
Odd Number	A number which is not divisible by 2. Odd numbers end in 1 ,3 ,5 ,7 or 9.
Operations	The four basic operations are addition, subtraction, multiplication and division.
Order of operations	The order in which operations should be done. BODMAS (see page 8)
Place value	The value of a digit dependent on its place in the number. Example: in the number 1573.4, the 5 has a place value of 100.
p.m.	(post meridiem) Any time in the afternoon or evening (between 12 noon and midnight).
Prime Factor	A factor of a number that also happens to be a prime number.
Prime Number	A number that has exactly 2 factors (can only be divided by itself and 1). Note that 1 is not a prime number as it only has 1 factor.
Qualitative	Qualitative data is data that is made up of words.

Quantitative	Quantitative data is data that is made up of numbers.
Quotient	What you get when one number is divided into another.
Remainder	The amount left over when dividing a number.
Reciprocal	The value given by dividing 1 by that number, or dividing that number INTO 1. Example: the reciprocal of 8 is $\frac{1}{8}$.
Share	To divide into equal groups.
Sum	The total of a group of numbers (found by adding).
Total	The sum of a group of numbers (found by adding).

Numeracy Timeline

The following table provides a guide to when a particular **Numeracy** topic is taught within the Broad General Education by the Maths Department at Lenzie Academy.

Month	S1	S2	S3
August	**Whole Numbers and Fractions of a Quantity**	**Whole Numbers and Integers**	*** Integers, Pythagoras**
September	**Decimals, Percentages of a Quantity and Integers**	**Decimals** (rounding, scientific notation, multiplying and dividing decimals), **Fractions and Percentages of Amounts.**	**Percentages and Money**
October	**Statistics and Money,** PROJECT:MANAGING A BUDGET	**Information Handling** (discrete and continuous data, mean, median, mode and range), **Financial Maths.** PROJECT: STOCKMARKET CHALLENGE	**Statistics** (stem and leaf, 5 figure summaries, box plots, semi-interquartile range)
November	**Money continued and Speed, Distance & Time** PROJECT:ST ANDREWS DAY	**Speed, Distance and Time**	**Speed, Distance and time**
December	PROJECT: MOVIE MAKER		
January	**Ratio and Proportion**	**Calculating Distances** (Pythagoras)	
February		**Area** (triangle, kite, rhombus and parallelogram), **Financial Maths.**	
March	**Fractions** (equivalent fractions, adding, subtracting and multiplying), **and Percentages**	**Fractions** (equivalent fractions, adding, subtracting, multiplying & dividing),	
April	PROJECT: FLOORING TASK		
May	**Probability and Mean, Median Mode and Range.**	**Information Handling** (scatter graphs, stem and leaf, probability), **Ratio and Proportion.**	
June	PROJECT:FAMOUS MATHEMATICIAN	**Financial maths** PROJECT:THEME PARK	

* Fractions, Scientific Notation, Significant Figures, Ratio & Proportion are completed in June when the timetable changes and S2 become S3.