Lenzie Academy Maths Department

National 5 Pupil Booklet

National 5: Expressions and Formulae

Learning Intention	I can simplify and carry out calculations using surds.			
Success Criteria		☺	☺	☹
• I know how to find the square, square root, cube or cube root of numbers. Evaluate 3^2 $\sqrt{49}$ 10^3 $\sqrt[3]{64}$				
• I can identify surds.				
• I know that $\sqrt{ab}=\sqrt{a}\times\sqrt{b}$, $\sqrt{a}\times\sqrt{b}=\sqrt{ab}$, $\sqrt{a}\times\sqrt{a}=a$ and $\sqrt{\dfrac{a}{b}}=\dfrac{\sqrt{a}}{\sqrt{b}}$.				
• I know how to fully simplify surds. Show that $\sqrt{75}=5\sqrt{3}$ and $\sqrt{72}=6\sqrt{2}$. Simplify $\sqrt{\dfrac{49}{100}}$				
• I can add and subtract surds. Simplify $2\sqrt{5}+7\sqrt{5}$, $\sqrt{75}-\sqrt{45}$ and $\sqrt{75}-\sqrt{27}$. Express $\sqrt{12}-\sqrt{3}+\sqrt{48}$ as a surd in its simplest form.				
• I can multiply surds. Expand and simplify $\sqrt{3}(\sqrt{3}-1)$ $\sqrt{2}(3-\sqrt{6})$ $(2+\sqrt{2})(3+\sqrt{2})$ $(2\sqrt{5})(2\sqrt{5}-1)$				
• I know how to rationalise the denominator of a fraction of the form $\dfrac{a}{\sqrt{b}}$. Express $\dfrac{3}{\sqrt{5}}$ with a rational denominator.				
EXTENSION • I know how to rationalise the denominator of a fraction of the form $\dfrac{a}{b\pm\sqrt{c}}$. Express $\dfrac{3}{1+\sqrt{2}}$ with a rational denominator.				

Learning Intention	I can simplify and evaluate expressions using the laws of indices.			

Success Criteria		☺	☺	☹
• I know that $3^4 = 3 \times 3 \times 3 \times 3$ and 3 is the base number and 4 is the index.				
• I know that $a^m \times a^n = a^{m+n}$	Simplify $\quad x^4 \times x^5 \qquad 3x^7 \times 5x^2$			
• I know that $a^m \div a^n = a^{m-n}$	Simplify $\quad x^8 \div x^5 \qquad x^2 \div x^{-3}$			
• I know that $(a^m)^n = a^{mn}$	Simplify $\quad (2a^3)^4$			
• I know that $a^0 = 1$	Simplify $\quad 5^0 \qquad (3ab^2)^0$			
• I know that $a^{-n} = \dfrac{1}{a^n}$	Rewrite with positive indices $\quad x^{-2} \qquad 3y^{-4}$			
• I know that $\dfrac{1}{a^{-n}} = a^n$	Rewrite with a positive indice $\quad \dfrac{2}{a^{-3}}$			
• I know that $a^{\frac{1}{n}} = \sqrt[n]{a}$	Evaluate $\quad 125^{\frac{1}{3}} \qquad 81^{-\frac{1}{2}}$			
• I know that $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$	Evaluate $\quad 16^{\frac{3}{4}} \qquad 8^{-\frac{2}{3}}$			
• I can simplify expressions of the form	$\dfrac{x^5 \times x^4}{x^{-2}} \qquad 6x^2 \times 2x^{-\frac{1}{3}} \qquad \sqrt{x}(x^3 - \dfrac{2}{x}) \qquad \sqrt[3]{a}(\sqrt[3]{a} - \dfrac{1}{\sqrt[3]{a}})$			

Learning Intention	I can carry out calculations using scientific notation.			

Success Criteria	☺	😐	☹
• I can write large and small numbers in scientific notation. $1820000 = 1{\cdot}82\times10^{6}$ $\qquad$ $0{\cdot}00049 = 4{\cdot}9\times10^{-4}$			
• I can carry out calculations using scientific notation. $\qquad$ Calculate $\quad (1{\cdot}2\times10^{5})\times(9\times10^{7})$			
• I can use my calculator to carry out calculations using values in scientific notation. There are 5×10^{9} red blood cells in 1 millilitre of blood. The average person has 5·5 litres of blood. How many red blood cells does the average person have in their blood? Give your answer in scientific notation.			
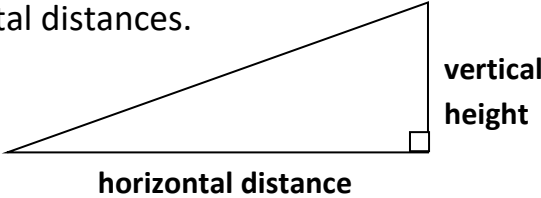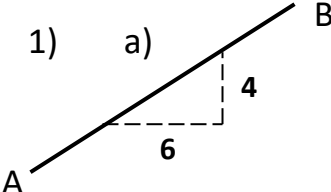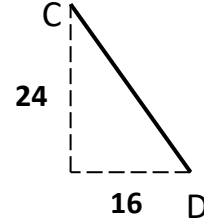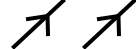
Learning Intention	I can simplify algebraic expressions involving the expansion of brackets.			

Success Criteria	☺	😐	☹
• I know how to expand a bracket and simplify: $\quad 3+4(b-2)$ $\qquad$ $4c-(c-3)$ $\qquad$ $4(2t+1)+5(3t-2)$			
• I know how to expand a bracket of the form: $\quad 2t(3t+1)$ $\qquad$ $7g(6-g)$			
• I know how to expand pairs of brackets with 2 linear expressions: $(x+3)(x+5)$ $\qquad$ $(4y+1)(3y-2)$ $\quad (3x-4)^{2}$			
• I know how to expand brackets with a linear and a quadratic expression: $\qquad (4y+1)(3y^{2}+5y-2)$			
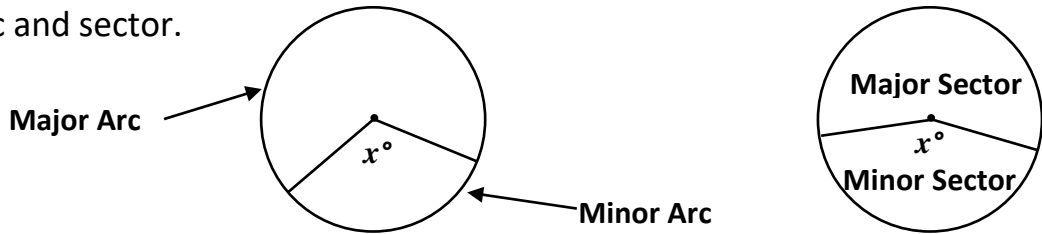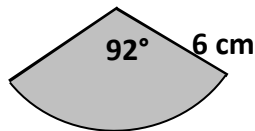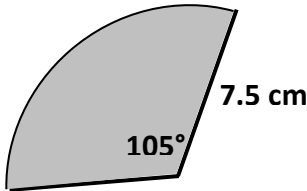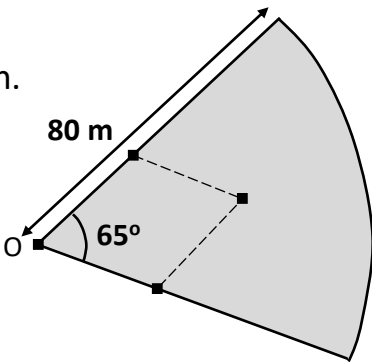
Learning Intention	I can factorise an algebraic expression.			
Success Criteria		☺	☺	☹
• I can factorise an expression by finding the Highest Common Factor (HCF). Factorise the following: $\quad 21 - 35x \qquad 8a^2b - 12ac$				
• I know how to factorise an expression using a difference of two squares. Factorise the following: $\quad x^2 - y^2 \qquad t^2 - 36 \qquad 9x^2 - y^2 \qquad 64 - 49y^2$				
• I know how to factorise an expression using a common factor and a difference of two squares. Factorise the following: $\quad 5x^2 - 20y^2$				
• I know that a trinomial expression is of the form $ax^2 + bx + c$.				
• I know how to factorise a trinomial expression of the form $x^2 + bx + c$. Factorise the following: $\quad x^2 + 6x + 8 \qquad x^2 - x - 6 \qquad x^2 + 5x - 6 \qquad x^2 - 5x - 6$				
• I know how to factorise a trinomial expression of the form $ax^2 + bx + c$. Factorise the following: $\quad 2x^2 + 7x + 3 \qquad 3x^2 - 10x - 8 \qquad 3x^2 - 16x + 5$				

Learning Intention	I can complete the square in a quadratic expression with unitary x^2 coefficient.			
Success Criteria		☺	☺	☹
• I know how to express $x^2 + bx + c$ in the form $(x + p)^2 + q$. Express $x^2 + 6x - 2$ and $x^2 - 3x + 4$ in the form $(x + p)^2 + q$.				
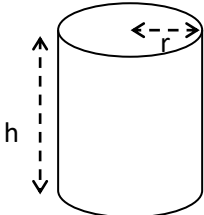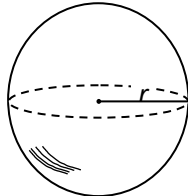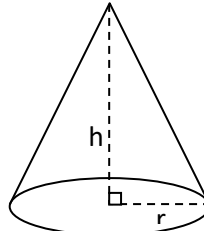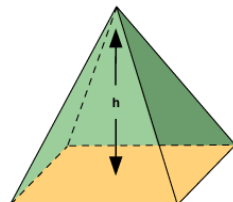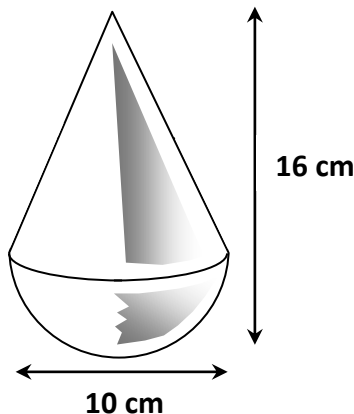
Learning Intention	I can reduce an algebraic fraction to its simplest form.			
Success Criteria		☺	😐	☹
• I can simplify fractions. Simplify the following: $\dfrac{7}{21}$ $\dfrac{27}{63}$				
• I can simplify algebraic fractions. Simplify the following: $\dfrac{x^2}{x^5}$ $\dfrac{10y^7}{15y^4}$ $\dfrac{(y+2)(y-3)}{(y-3)(y-4)}$ $\dfrac{x^2-4}{2x+4}$				

Learning Intention	I can carry out calculations with algebraic fractions.			
Success Criteria		☺	😐	☹
• I can add, subtract, multiply and divide fractions. Evaluate $3\dfrac{2}{5}+1\dfrac{1}{3}$, $2\dfrac{3}{4}\times1\dfrac{1}{5}$ and $2\dfrac{1}{3}\div1\dfrac{3}{4}$.				
• I can add and subtract algebraic fractions. Simplify the following: $\dfrac{x}{2}-\dfrac{x}{3}$, $\dfrac{5}{x}+\dfrac{2}{y}$, $\dfrac{t}{x}-\dfrac{3}{y}$ and $\dfrac{x+1}{2}+\dfrac{x-1}{3}$.				
• I can multiply and divide algebraic fractions. Simplify the following: $\dfrac{t}{5}\times\dfrac{3}{y}$, $\dfrac{t}{15}\times\dfrac{25}{t^2}$ and $\dfrac{x}{7}\div\dfrac{x^3}{14}$.				

Learning Intention	I can calculate the gradient of a straight line, given two points.			

Success Criteria	☺	☺	☹
• I can calculate the gradient of a line using vertical and horizontal distances. $$\text{Gradient} = \frac{\text{vertical height}}{\text{horizontal distance}}$$ **vertical height** **horizontal distance** Find the gradient of these lines: 1) a) B b) C 4 6 A 24 16 D			
• I can recognise lines with positive ╱ and negative ╲ gradients.			
• I can recognise lines with zero ── and undefined │ gradients.			
• I know that parallel lines have equal gradients. ↗ ↗			
• I know that the gradient formula is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.			
• I know how to use the gradient formula. Calculate the gradient of the line joining $A(1,-7)$ and $B(4,3)$. Calculate the gradient of the line joining $C(2,-3)$ and $D(8,-3)$. Calculate the gradient of the line joining $E(4,5)$ and $F(4,3)$.			

Learning Intention I can calculate the length of an arc and the area of a sector of a circle.			
Success Criteria	☺	😐	☹
• I can calculate the circumference and area of a circle using $C = \pi d$ and $A = \pi r^2$.			
• I know the meaning of arc and sector. **Major Arc** $x°$ **Minor Arc** **Major Sector** $x°$ **Minor Sector**			
• I know how to calculate the length of an arc using arc length = $\dfrac{x}{360} \times \pi d$. Calculate the length of the arc shown. 92° 6 cm			
• I know how to calculate the area of a sector using sector area = $\dfrac{x}{360} \times \pi r^2$. Calculate the area of the sector of the circle shown. 7.5 cm 105°			
A school baseball field is in the shape of a sector of a circle as shown. Given that O is the centre of the circle, calculate: (a) the perimeter of the playing field (b) the area of the playing field. 80 m 65° O			

Learning Intention	I can calculate the volume of a standard solid rounding my answer appropriately.			
Success Criteria		☺	☺	☹

- I can calculate the volume of any solid given its formula.

cylinder	sphere	cone	pyramid

$$V = \pi r^2 h$$

$$V = \frac{4}{3}\pi r^3$$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}Ah$$

The football has a diameter of 30 cm.

Calculate its volume, take $\pi = 3\cdot14$.(non-calculator example)

30 cm

- I can solve problems rounding my final answer using significant figures.

A child's toy is in the shape of a hemisphere with a cone on top, as shown.

The toy is 10 cm wide and 16 cm high. Calculate the volume of the toy.

Give your answer correct to 2 significant figures.

16 cm

10 cm

National 5: Relationships
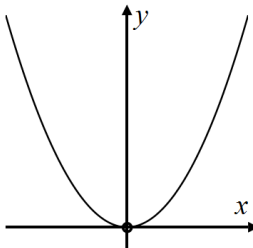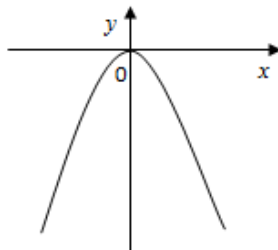
Learning Intention	I can use and interpret straight line equations.			

Success Criteria	☺	☺	☹
• I can use and interpret the straight line equation $y = mx + c$. (1) Write down the gradient of the line $y = 2x - 4$ and the coordinates of the point where it crosses the *y*-axis. (2) Sketch the lines with equation $y = -x + 3$, $y = -5$ and $x = 4$. (3) Find the equation of the straight lines shown in the diagram. (4) Write down the gradient and the y-intercept of the line $2x + 3y = 6$.			
• I know that $y - b = m(x - a)$ represents a straight line with gradient m, passing through the point (a, b).			
• I can determine the equation of a straight line using $y - b = m(x - a)$. Find the equation of the straight lines which pass through the point: (a) $(1,5)$ with a gradient of 2 (b) $(-4,3)$ with a gradient of $\dfrac{2}{5}$			
• I can determine the equation of a straight line using two points which lie on the line. Find the equation of the line joining $A(-2,-3)$ and $B(8,2)$.			
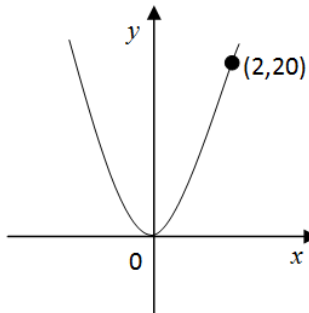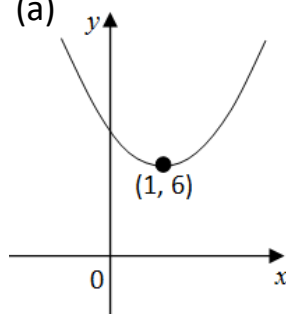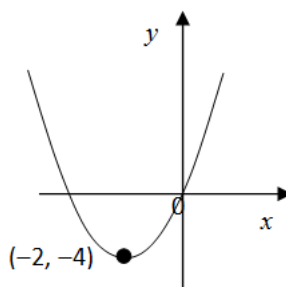
Learning Intention	I can use functional notation.			
Success Criteria		☺	😐	☹
• I know that functional notation can be expressed as $f(x), g(x), h(t)$.....				
• I can evaluate an expression in functional notation. A function is defined as $f(x) = x^2 - 3$, find the value of $f(x)$ when $x = 4$.				
• I can calculate x given the value of $f(x)$. A function is defined by $f(x) = 8 - 3x$. Find x when $f(x) = -13$. A function is defined by $f(t) = t^2 - 1$. Find the **values** of t when $f(t) = 8$.				

Learning Intention	I can solve linear equations and inequations.			
Success Criteria		☺	😐	☹
• I can solve linear equations. Solve $3x + 5 = 17$ $8x - 11 = 5$ $5x - 2 = 2x + 23$ $7x + 11 = 4x - 19$				
• I can solve equations involving brackets. Solve $3(x - 5) = 21$ $5(x + 7) - 2(3x - 4) = 45$ $x(x + 3) = x^2 + 15$ $(x - 1)^2 + 7^2 = x^2$				
• I can solve inequations. Solve $5x + 3 < 12$ $7x - 2 > 10x + 4$ $10 - 2(x + 3) > 3(x - 2)$				
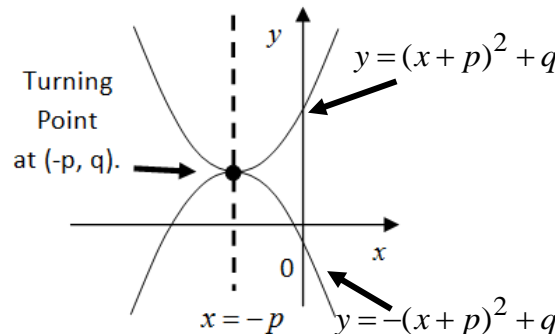
Learning Intention	I can solve problems using simultaneous linear equations.			
Success Criteria		☺	😐	☹

• I know how to solve systems of linear equations graphically. Use the diagram below to solve $x + 2y = 8$ and $3x + 2y = 12$.				
• I know how to solve systems of equations algebraically using **substitution** or **elimination**. Solve **algebraically** the system of equations (a) $\begin{array}{l} 3x + y = 10 \\ 5x - 2y = 13 \end{array}$ (b) $\begin{array}{l} 3x - 2y = 11 \\ 2x + 5y = 1 \end{array}$				
• I know how to create and solve systems of equations algebraically. Seats on flights from London to Edinburgh are sold at two prices, £30 and £50. On one flight a total of 130 seats were sold. Let x be the number of seats sold at £30 and y be the number of seats sold at £50. (a) Write down an equation in x and y which satisfies the above condition. The sale of the seats on this flight totalled £6000. (b) Write down an equation in x and y which satisfies this condition (c) How many seats were sold at each price?				

Learning Intention	I can change the subject of a formula.			

Success Criteria	☺	☺	☹
• I recognise formulae that can be rearranged in 1 step when changing the subject to x. $x + A = B$ $gx = k$ $\dfrac{x}{t} = f$			
• I recognise formulae that can be rearranged in 2 steps or more when changing the subject to x. $dx - h = k$ $\dfrac{d}{x} = g$ $y = \dfrac{7x}{3} - 4$			
• I can rearrange formulae involving squares and square roots Change the subject of : $V = \pi r^2 h$ to r $E = \dfrac{1}{2}mv^2$ to v $r = \sqrt{\dfrac{A}{\pi}}$ to A $s = \sqrt{\dfrac{t}{k}}$ to k $gh = \dfrac{(x - 3y)}{A^2}$ to A $b^2 = \sqrt{d} - 4$ to d.			

Learning Intention	I can recognise a quadratic function from its graph.			
Success Criteria		☺	😐	☹
• I can recognise and draw $y = x^2$ and $y = -x^2$				

Learning Intention I can recognise and determine the equation of a quadratic function from its graph.				
Success Criteria		☺	😐	☹
• I know how to identify the value of a from the graph of $y = ax^2$. The graph with equation $y = ax^2$ is shown. The point (2, 20) lies on the graph. Determine the value of a.				
• I can identify the values of p and q from the graph of $y = (x + p)^2 + q$. The two diagrams show graphs of $y = (x + p)^2 + q$. Write down the values of p and q. (a) (b)				
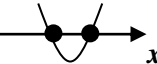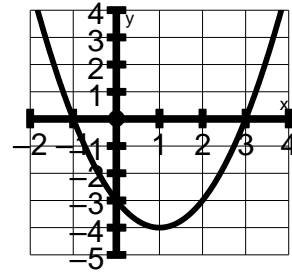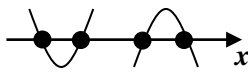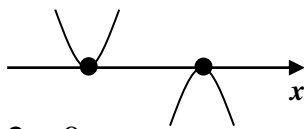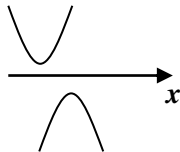
Learning Intention	I can identify the main features and sketch a quadratic function of the form $y = (x - m)(x - n)$.			
Success Criteria		☺	😐	☹
• I can identify the roots and y-intercept of $y = (x - m)(x - n)$. Find the roots and y-intercept of $\quad y = (x - 1)(x - 5)$ and $y = (x - 3)(x + 4)$.				
• I can find the equation of the axis of symmetry and the coordinates and nature of the turning point of $y = (x - m)(x - n)$. Find the equation of the axis of symmetry and the coordinates and nature of the turning point of $y = (x - 1)(x - 5)$ and $y = (x - 3)(x + 4)$.				
• I can sketch and annotate $y = (x - m)(x - n)$. Sketch the graph $y = (x - 4)(x + 2)$ on plain paper showing clearly where the graph crosses the axes and state the coordinates and nature of the turning point.				
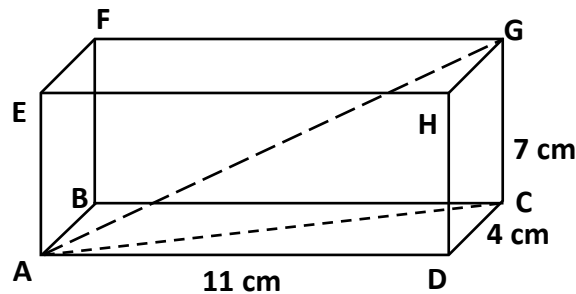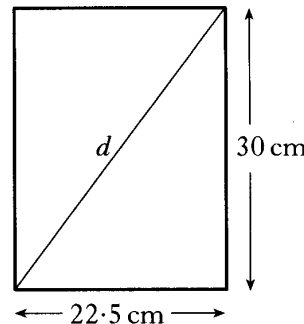
Learning Intention	I can identify the main features and sketch a quadratic function of the form $$y = (x + p)^2 + q \text{ and } y = -(x + p)^2 + q \text{ or } y = q - (x + p)^2.$$			
Success Criteria		☺	😐	☹
• I know that $y = (x + p)^2 + q$ has a **minimum** value of q when $x = -p$. Hence the **minimum turning point** is at $(-p, q)$ and $x = -p$ is the equation of the axis of symmetry.	Turning Point at (-p, q).			
• I know that $y = -(x + p)^2 + q$ or $y = q - (x + p)^2$ has a **maximum** value of q when $x = -p$. Hence the **maximum turning point** is at $(-p, q)$ and $x = -p$ is the equation of the axis of symmetry.	$x = -p$ $\quad y = -(x + p)^2 + q$			
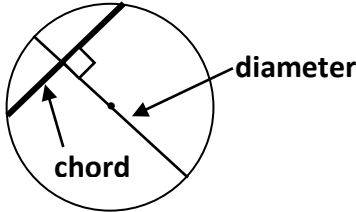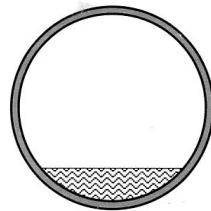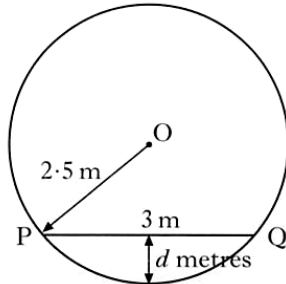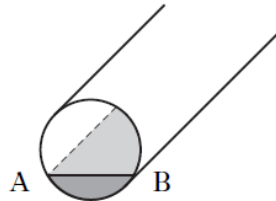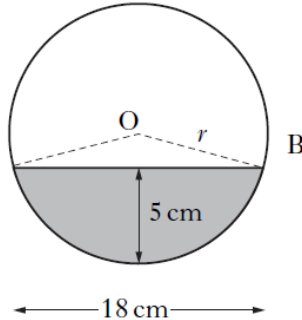
Success Criteria	☺	😐	☹
• I can identify the equation of the axis of symmetry and the coordinates and nature of the turning point of $y = (x+p)^2 + q$ and $y = -(x+p)^2 + q$ or $y = q - (x+p)^2$. The equation of the parabola in the diagram is $y = (x-2)^2 - 7$ (a) State the coordinates of the minimum turning point of the parabola. (b) State the equation of the axis of symmetry of the parabola.			
• I can sketch and annotate $y = (x+p)^2 + q$ and $y = -(x+p)^2 + q$ or $y = q - (x+p)^2$. A parabola has equation (a) $y = (x-4)^2 + 9$ (b) $y = 11 - (x+2)^2$. For each example (i) State the equation of the axis of symmetry. (ii) Write down the coordinates of the turning point stating whether it is a maximum or minimum. (iii) Make a sketch of the function.			
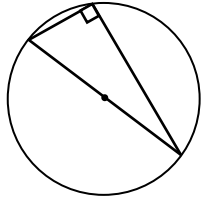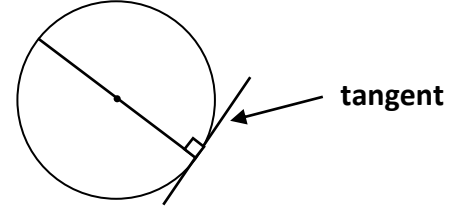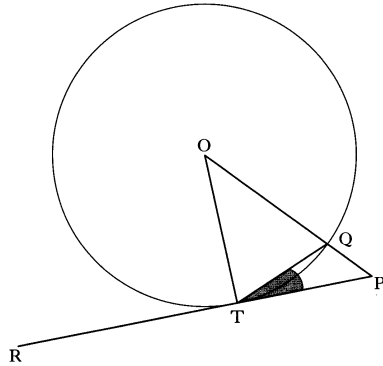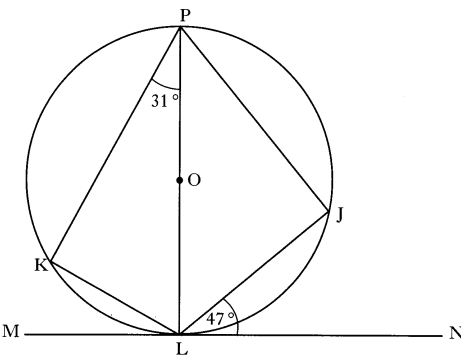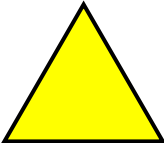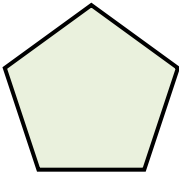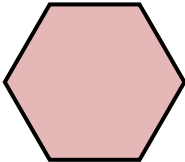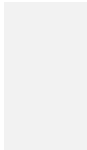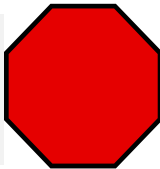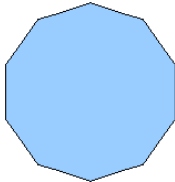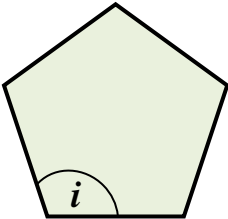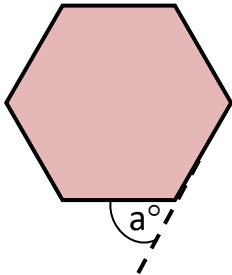
Learning Intention	I can solve quadratic equations.			

Success Criteria	☺	☺	☹
• I know that a quadratic equation is of the form $y = ax^2 + bx + c$ where $a \neq 0$.			
• I know the meaning of root.			
• I know that to solve a quadratic equation it must be of the form $ax^2 + bx + c = 0$.			
• I can solve a quadratic equation graphically. The diagram shows the graph of the function $y = x^2 - 2x - 3$. Use the graph to solve the equation $x^2 - 2x - 3 = 0$.			
• I can solve a quadratic equation using factorisation. Solve the equation $x^2 - x - 12 = 0$.			
• I can solve a quadratic equation using the quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Solve the equation $2x^2 + 3x - 1 = 0$ using the quadratic formula giving your answers correct to one decimal place.			
• I know that the value of the discriminant "$b^2 - 4ac$" determines the nature of the roots of a quadratic equation: If $b^2 - 4ac > 0$ the roots are real and unequal/distinct If $b^2 - 4ac = 0$ the roots are real and equal If $b^2 - 4ac < 0$ there are no real roots. (1) Find the nature of the roots of $x^2 - x - 12 = 0$. (2) Find the values of k for which the equation $2x^2 + 4x + k = 0$ has equal roots.			

Learning Intention	I can use and apply the Theorem of Pythagoras.			
Success Criteria		☺	😐	☹
• I can solve problems by applying the Theorem of Pythagoras to 2D and 3D shapes by identifying and drawing a right angled triangle and labelling the sides appropriately. In the cuboid shown opposite. (a) Calculate the length of the face diagonal AC. (b) Hence calculate the length of the space diagonal AG.				
• I know when to use the converse of the Theorem of Pythagoras.				
• I know how to use the converse of the Theorem of Pythagoras and can communicate my solution and conclusion correctly. A rectangular picture frame is to be made. It is 30 centimetres high and 22·5 centimetres wide, as shown. To check that the frame is rectangular, the diagonal, d, is measured. It is 37·3 centimetres long. Is the frame rectangular?				

Learning Intention	I can solve problems involving chords in circles, often using Pythagoras.			

Success Criteria		☺	😐	☹
• I know that a chord is a line joining two points on the circumference of a circle.				
• I know that the diameter is a special chord passing through the centre of a circle.				
• I know that, at the point of contact, a chord is perpendicular to the radius or diameter of a circle.	diameter chord			

(1) The diagram shows a circular cross-section of a cylindrical oil tank.

In the figure opposite.

➢ O represents the centre of the circle
➢ PQ represents the surface of the oil in the tank
➢ PQ is 3 metres
➢ the radius OP is 2·5 metres

Find the depth, d metres, of oil in the tank.

(2) A pipe has water in it as shown.

➢ The depth of the water is 5 centimetres.
➢ The width of the surface, AB, is 18 centimetres.

Calculate, r, the radius of the pipe.

Learning Intention	I can determine an angle involving at least two steps.			

Success Criteria	☺	😐	☹
• I know that every triangle in a semi-circle is right angled.			
• I know that a tangent is a straight line which touches a circle at one point only. • I know that, at the point of contact, a tangent is perpendicular to the radius or diameter of a circle. **tangent**			

(1)   RP is a tangent to the circle; centre O, with a point of contact at T.
The shaded angle PTQ = 24°. Calculate the sizes of angle OPT.

(2)   The tangent, MN, touches the circle, centre O, at L.
Angle JLN = 47° Angle KPL = 31°
Find the size of angle KLJ.

Success Criteria	☺	😐	☹
• I know that a polygon is a many sided shape.			
• I can name the following regular polygons: • I know how to find the sum of the angles inside any polygon.			
• I know that interior angles are the angles inside a polygon. • I know that exterior angles are formed by extending one side of a polygon as shown in the diagram. • I know that interior angle + exterior angle = 180°. i = interior angle e = exterior angle			

• I know how to determine the value of an interior and an exterior angle for any regular polygon.
 	
(1) Here is a regular pentagon.
    Calculate the size of angle $i°$.
 	
(2) Here is a regular hexagon.
    Calculate the size of angle a°.			

Learning Intention	I can solve problems involving similarity.			

Success Criteria	☺	☻	☹
• I know that similar shapes are equiangular and that their corresponding sides are in the same ratio.			
• I know how to find a linear scale factor.			
• I can solve problems using a linear scale factor. The diagram shows the design for a house window. Find the value of x.			
• I know how to find an area scale factor.			
• I can solve problems using an area scale factor. These shapes are mathematically similar. The area of the larger shape is 84 cm². Calculate the area of the smaller shape.			
• I know how to find a volume scale factor.			
• I can solve problems using a volume scale factor. These solid shapes are mathematically similar. The volume of the smaller shape is 20 mm³. Calculate the volume of the larger shape			

Diagram labels: 1·0 m, 1.2 m, 0·5 m, x m

Area shapes: 12 cm, 84 cm², 6 cm

Volume shapes: 20 mm³, 2 mm, 6 mm
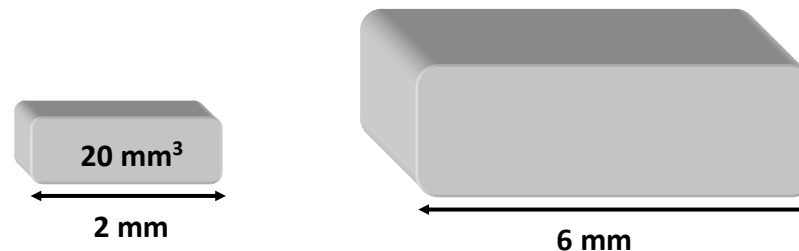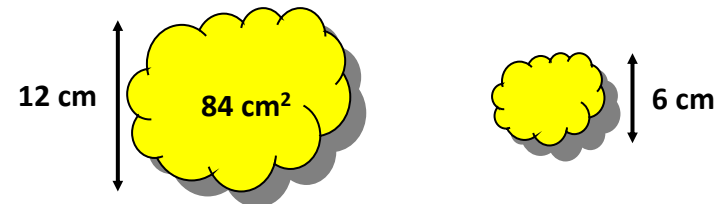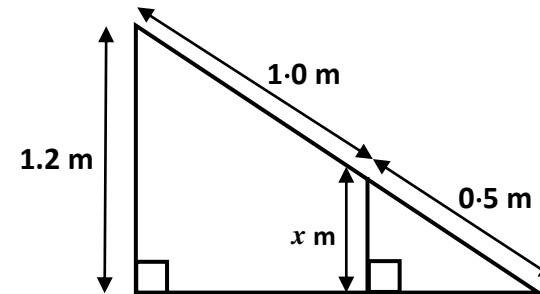
Learning Intention	I can interpret and sketch trigonometric graphs.			
Success Criteria		☺	☺	☹

- I can recognise and sketch:
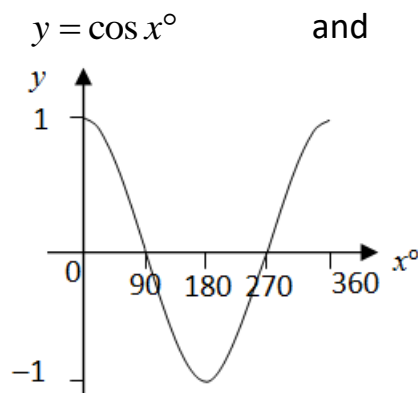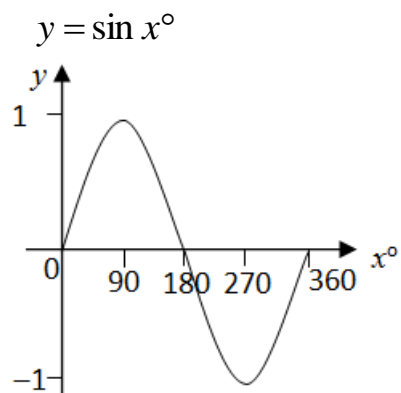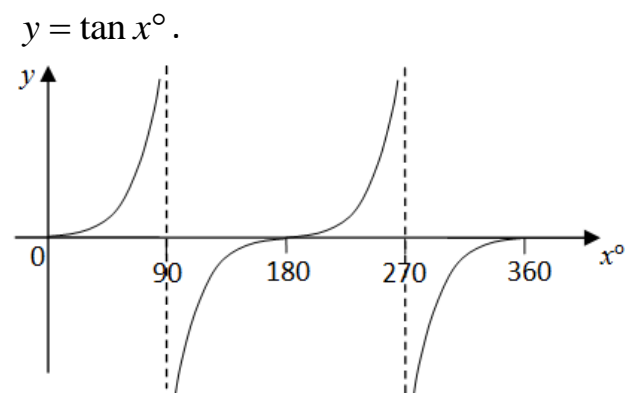
  $y = \sin x°$ $y = \cos x°$ and $y = \tan x°$.

- I know the value of $y = \sin x°$, $y = \cos x°$ and $y = \tan x°$ at 0°, 90°, 180°, 270° and 360° .

- I know the meaning of amplitude, period, vertical translation and phase angle.

- I can identify and sketch the graph of $y = \sin(x \pm a)°$ and $y = \cos(x \pm a)°$.

(1)   Write down the equation for each graph.

(a)

(b)
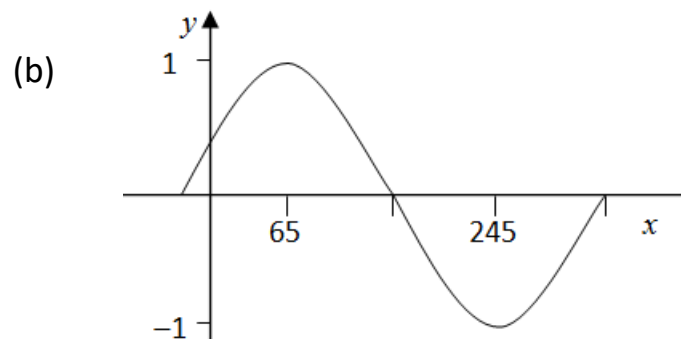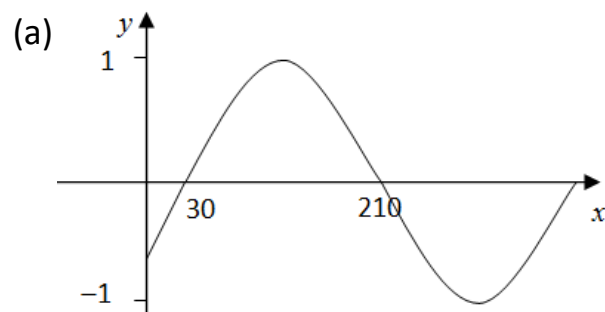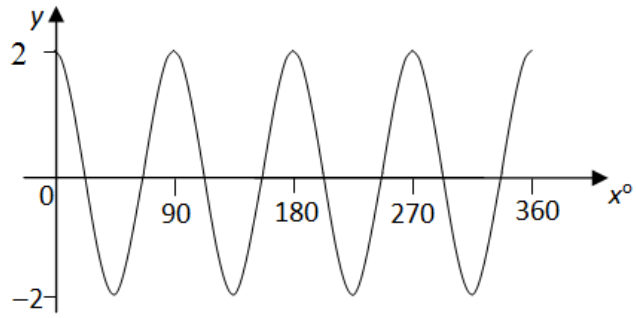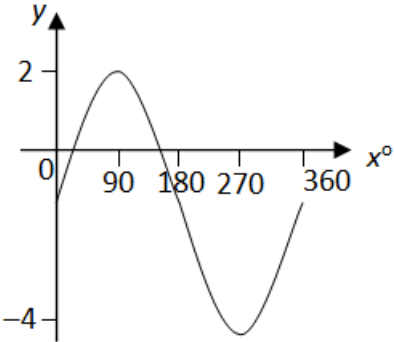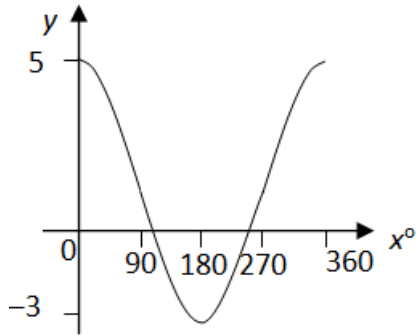
(2) Make a neat sketch of these trigonometric functions showing the important values for $0° \le x \le 360°$.
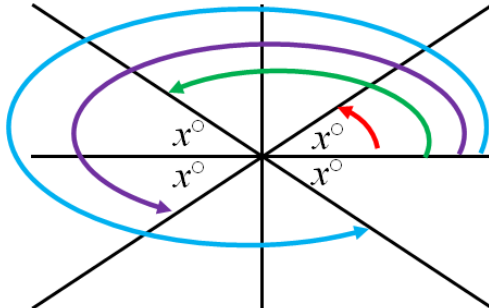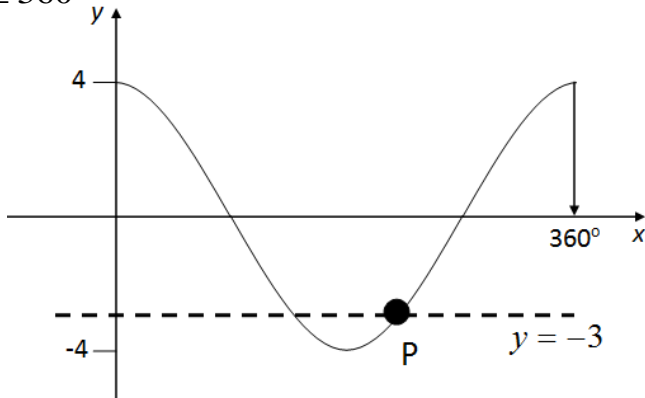
(a) $y = \cos(x - 60)°$ (b) $y = \sin(x + 30)°$ (c) $y = \cos(x - 90)°$
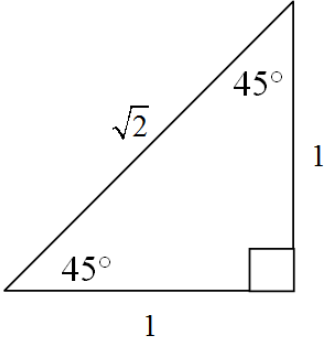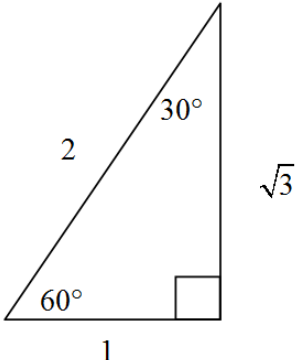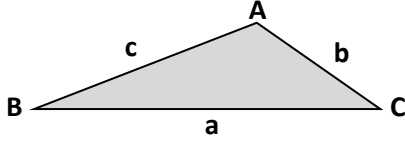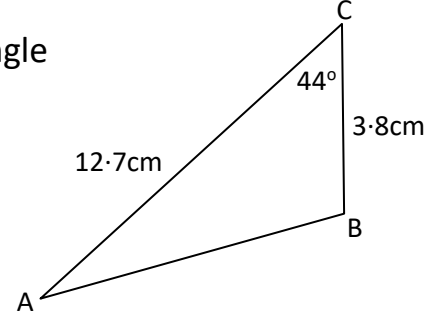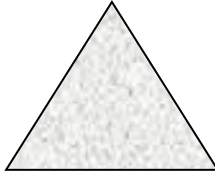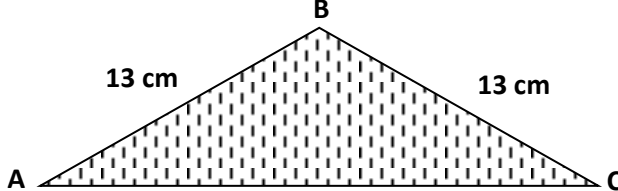
- I can identify and sketch the graph of $y = a\sin bx°$ and $y = a\cos bx°$.

(1) Part of the graph of $y = a\cos bx°$ is shown in the diagram.

State the values of a and b.

(2) Identify the maximum value, minimum value and period of $y = 5\sin 3x°$.

- I can identify and sketch the amplitude, period and vertical translation from the graph of $y = a\sin bx° + c$ and $y = a\cos bx° + c$

(1)   Determine the amplitude, period and equation for each graph.   (a)

(b)

(2) Make sketches of the following functions for $0° \le x \le 360°$, clearly marking any important points.

(a) $y = 3\cos x° + 2$ (b) $y = 4\sin x° - 5$ (c) $y = 5\sin 4x° + 6$
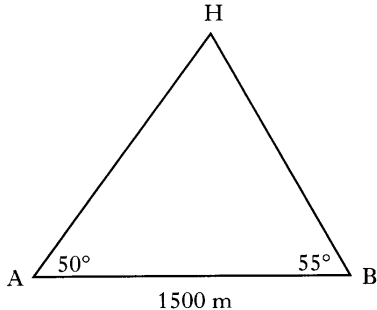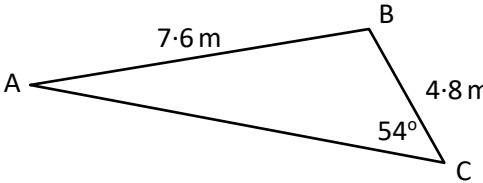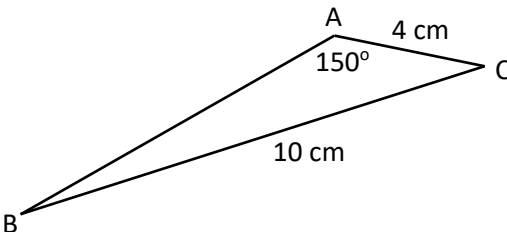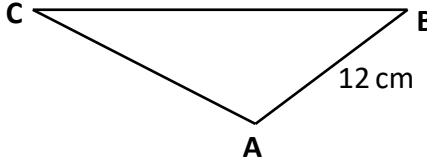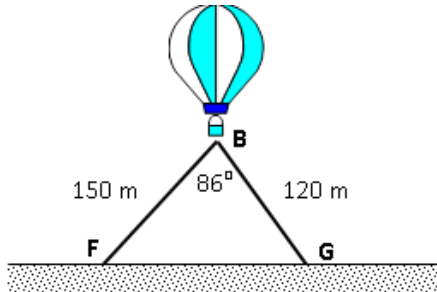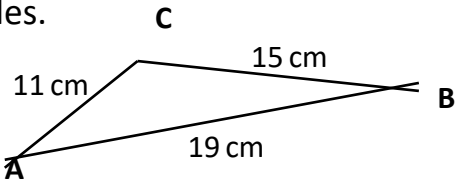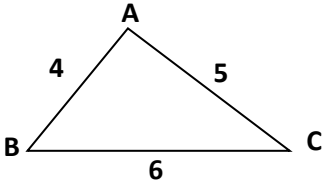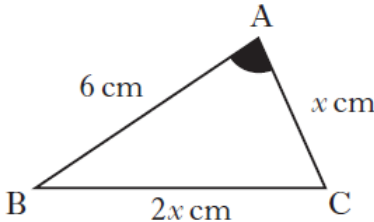
Learning Intention	I can solve trigonometric equations.			

Success Criteria	☺	☺	☹
• I know when $y = \sin x°$, $y = \cos x°$ and $y = \tan x°$ are positive or negative in value.			
• I can use a quadrant diagram to find related angles. **SIN Positive** | **All Positive** Related angle $= 180 - x°$ | Basic angle $= x°$ **TAN Positive** | **COS Positive** Related angle $= 180 + x°$ | Related angle $= 360 - x°$			

• I can solve trigonometric equations.
(1)   Solve   (a) $\cos x° = 0\cdot 5$ (b) $3\sin x° - 2 = 0$ for $0° \le x \le 360°$
 	
(2) The graph in the diagram has an equation of the form $y = a\cos x°$.
   (a) The broken line in the diagram has equation $y = -3$.
   (b) Determine the coordinates of the point P.			
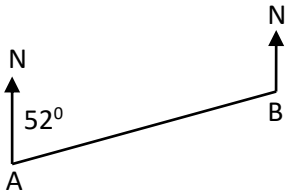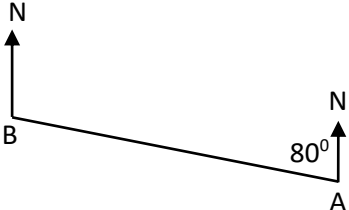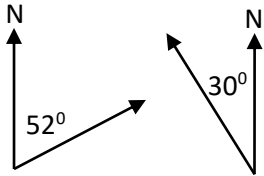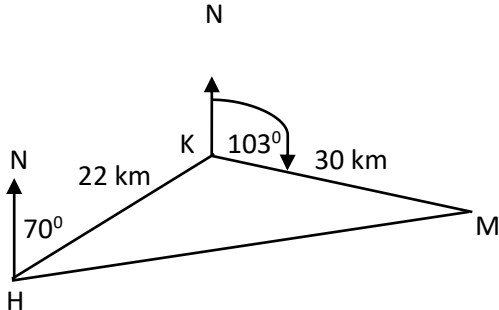
Learning Intention	I can work with exact values and trigonometric identities.			

Success Criteria	☺	☺	☹
• I know the exact values of $y = \sin x°$, $y = \cos x°$ and $y = \tan x°$ at 30°, 45° and 60° using these two triangles.			
• I can calculate the **exact** value of obtuse and reflex angles from their related angles. Determine the **exact** value of (a) cos150° (b) sin 240° (c) tan 315°.			
• I can simplify trigonometric expressions using the trigonometric identities $\sin^2 x + \cos^2 x = 1$ and $\tan x = \dfrac{\sin x}{\cos x}$. (a) Show that $\dfrac{1 - \cos^2 x}{\cos^2 x} = \tan^2 x$ (b) Simplify $\cos x \tan x$ (c) Prove that $3\sin^2 \theta + 2\cos^2 \theta = 2 + \sin^2 \theta$.			
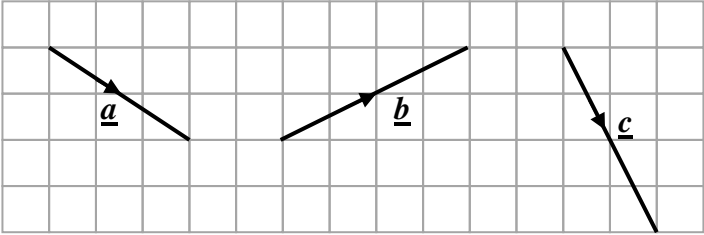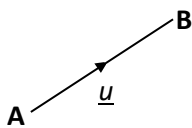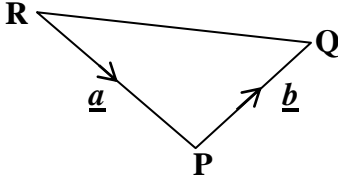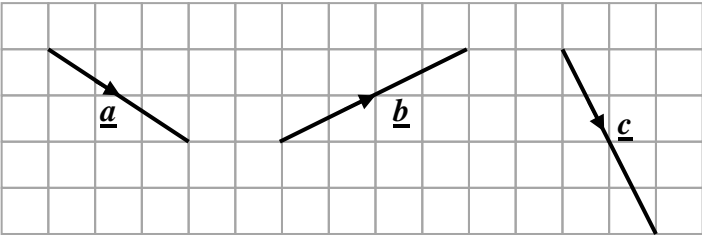
National 5: Applications

Learning Intention	I can calculate the area of a triangle using trigonometry.			
Success Criteria		☺	😐	☹
• I can draw and label the sides and angles of any triangle. In any triangle I know that the **largest angle** is opposite the **longest side**. In any triangle I know that the **smallest angle** is opposite the **shortest side**.				
• I know how to use the area rule, $Area = \dfrac{1}{2}ab\sin C$, to calculate the area of any triangle given two sides and the included angle. Calculate the area of the triangle shown giving your answer to 3 significant figures.				
• Mr Fields is planting a rose-bed in his garden. It is to be in the shape of an **equilateral triangle** of side 2m. What area of lawn will he need to remove to plant his rose-bed?				
• The area of a triangular napkin is 80·4 cm². Calculate the size of the **obtuse** angle ABC.				

Learning Intention	I can use the sine rule to find a side or angle.			

Success Criteria	☺	😐	☹
• I know how to use the sine rule, $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}$ to find a side. A helicopter, at point H, hovers between two aircraft carriers at points A and B which are 1500 metres apart. From carrier A, the angle of elevation of the helicopter is 50º. From carrier B, the angle of elevation of the helicopter is 55º. Calculate the distance from the helicopter to the **nearest carrier**.			
• I know how to use the sine rule, $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}$, to find an angle. Calculate the size of angle BAC in this triangle.			
• In triangle ABC: AC = 4 centimetres, BC = 10 centimetres and Angle BAC =150^0. Given that sin 150^0= $\dfrac{1}{2}$, show that sin B = $\dfrac{1}{5}$.			
• In triangle ABC, AB = 12 cm, sin C = $\dfrac{1}{2}$ and sin B = $\dfrac{1}{3}$. Find the length of side AC.			

Learning Intention	I can use the cosine rule to find a side or angle.			

Success Criteria		☺	☺	☹
• I know how to use the cosine rule, $a^2 = b^2 + c^2 - 2bc\cos A$, to find a side given 2 sides and the included angle. A hot air balloon B is fixed to the ground at F and G by 2 ropes 150m and 120 m long. If $\angle FBG$ is 86º, how far apart are F and G?				
• I know how to use the cosine rule, $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$, to find an angle given all 3 sides. Calculate the size of angle ABC.				
• In triangle ABC, AB = 4 units, AC = 5 units and BC = 6units. Show that $\cos A = \dfrac{1}{8}$.				
• In triangle ABC: cos A =0·5, AB = 6 centimetres, BC = 2x centimetres and AC = x centimetres. Show that $x^2 + 2x - 12 = 0$.				

Learning Intention	I can solve trigonometry problems with bearings.			

Success Criteria	☺	😐	☹
- I know that a bearing is measured from a North line in a **clockwise** direction. - I know that North has a bearing of 000°. (1) Write down the 3 figure bearing to represent these angles. (2) In each of the following write down the 3 figure bearing of: (a) B from A and (b) A from B. 			
I know to draw and annotate a triangle to illustrate a problem. I know to draw North lines in order to find angles.			
I can solve problems by applying the sine and cosine rules. In the diagram shown three towns, Holton, Kilter and Malbrigg are represented by the points H, K and M respectively. A helicopter flies from Holton for 22 kilometres on a bearing of 070^0 to Kilter. It then flies from Kilter for 30 kilometres on a bearing of $103°$ to Malbrigg. The helicopter then returns directly to Holton. (a) Calculate the size of angle HKM. (b) Calculate the total distance travelled by the helicopter. Do not use a scale drawing.			

Learning Intention	I can work with 2D vectors.			

Success Criteria	☺	☻	☹
• I know that a vector has magnitude and direction.			

• I know that a vector can be illustrated as a directed line segment and it can be named as $\overrightarrow{AB} = \underline{u}$

• I can add and subtract vectors using directed line segments.

The diagram shows 3 vectors $\underline{a}$, $\underline{b}$ and $\underline{c}$.
Draw representations of these vectors.
(a) $\underline{a} + \underline{b}$ (b) $2\underline{b} + \underline{c}$ (c) $-2\underline{a}$ (d) $3\underline{a} + 2\underline{b}$
(e) $\underline{b} - \underline{c}$ (f) $\underline{c} - \underline{a}$ (g) $\underline{a} + \underline{b} - \underline{c}$

• I can solve problems in a diagram with directed line segments.
Express each of the following displacements in terms of vectors $\underline{a}$ and $\underline{b}$.

(a) $\overrightarrow{PQ}$ (b) $\overrightarrow{QP}$ (c) $\overrightarrow{PR}$ (d) $\overrightarrow{RQ}$ (e) $\overrightarrow{QR}$

• I can write a 2D vector in component form $\begin{pmatrix} x \\ y \end{pmatrix}$.

Write the vectors $\underline{a}$, $\underline{b}$ and $\underline{c}$ in component form.

- I can add and subtract 2D vectors in component form and multiply 2D vectors in component form by a scalar.

  If $\underline{u} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ and $\underline{v} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ calculate in component form the value of :   (a) $\underline{u} + \underline{v}$ (b) $\underline{u} - \underline{v}$ (c) $3\underline{u} - 4\underline{v}$.

- I know that the magnitude is the length of a vector and that $\left|\underline{u}\right|$ represents the magnitude of vector $\underline{u}$.

- I know how to calculate the magnitude of a 2D vector. If $\underline{u} = \begin{pmatrix} x \\ y \end{pmatrix}$ then $\left|\underline{u}\right| = \sqrt{x^2 + y^2}$.

  If $\underline{u} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ and $\underline{v} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ calculate   (a) $\left|\underline{u}\right|$ (b) $\left|\underline{v}\right|$ (c) $\left|2\underline{u} + \underline{v}\right|$ (d) $\left|3\underline{u} - 4\underline{v}\right|$.

Learning Intention   I can work with 3D coordinates.

Success Criteria	☺	😐	☹
- I know that (x, y, z) represents the coordinates of a point in 3 dimensions. - I can determine the 3D coordinates of a point from a diagram. A **cube** of side 6 units is placed on coordinate axes as shown in the diagram. Write down the coordinates of each vertex of the cube.			

Learning Intention	I can work with 3D vectors.			
Success Criteria		☺	☻	☹
• I can write a 3D vector in component form $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$.				
• I can add and subtract 3D vectors in component form and multiply 3D vectors in component form by a scalar. If $\underline{u} = \begin{pmatrix} 4 \\ 5 \\ -2 \end{pmatrix}$ and $\underline{v} = \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}$ calculate in component form the value of: (a) $\underline{u} + \underline{v}$ (b) $2\underline{u} - \underline{v}$ (c) $3\underline{u} + 4\underline{v}$.				
• I know how to calculate the magnitude of a 3D vector. If $\underline{u} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ then $|\underline{u}| = \sqrt{x^2 + y^2 + z^2}$. If $\underline{u} = \begin{pmatrix} 4 \\ 5 \\ -2 \end{pmatrix}$ and $\underline{v} = \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}$ calculate (a) $|\underline{u}|$ (b) $|\underline{v}|$ (c) $|\underline{u} + 3\underline{v}|$ (d) $|2\underline{u} - 3\underline{v}|$				

Learning Intention	I can solve problems using reverse percentages.			

Success Criteria	☺	😐	☹
• I can recognise reverse percentages problems.			
• I know how to use reverse percentages to find the **original amount**. (1) A coat was reduced by 30% in a sale to £105 what was its original price? (2) A gym's membership has increased by 17% over the past year. It now has 585 members. How many members did it have a year ago?			
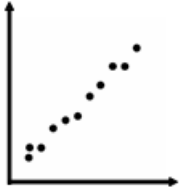
Learning Intention	I can solve appreciation and depreciation problems.			

Success Criteria	☺	😐	☹
• I know the meaning of appreciation and depreciation and can recognise appreciation and depreciation problems.			
• I can recognise compound interest problems.			

• I can solve appreciation, depreciation and compound interest problems.
(1) A house was bought for £80 000 3 years ago. It appreciated in value by 4% the first year, 7% the second
   and 11% the third. Calculate the value of the house after 3 years. Give your answer to 3 significant figures.
(2) A computer was bought for £999.
   If it depreciates in value by 18% per year when will its value be less than half its original price?
(3) David Smith buys a flat for £120 000.
   If it appreciates in value by 7% per year for 5 years how much is it worth after 5 years?
(4) Joseph invests £4500 in a bank that pays 6·4% interest per annum. If Joseph does not touch the money in the
   bank, how much interest will he have gained after 3 years? Give your answer to **the nearest penny**.			

Learning Intention	I can carry out calculations involving fractions.			

Success Criteria	☺	😐	☹
• I can recognise a mixed number and an improper fraction.			
• I can change any mixed number into an improper fraction. Write $3\frac{2}{5}$ as an improper fraction.			
• I can change any improper fraction into a mixed number. Write $\frac{27}{4}$ as a mixed number.			
• I can add and subtract fractions. Evaluate each of the following: (a) $\frac{2}{7}+\frac{1}{8}$ (b) $\frac{1}{6}+\frac{3}{5}$ (c) $\frac{7}{9}-\frac{3}{7}$ (d) $4\frac{2}{3}+3\frac{1}{12}$ (e) $8\frac{2}{5}-1\frac{3}{10}$			
• I can multiply and divide fractions. Evaluate each of the following: (a) $\frac{5}{7}\times\frac{14}{15}$ (b) $2\frac{1}{4}\times3\frac{1}{2}$ (c) $\frac{3}{7}\div\frac{11}{14}$ (d) $3\frac{3}{5}\div2\frac{1}{4}$ (e) $3\frac{1}{3}\times1\frac{1}{8}\times8\frac{1}{3}$			
• I can apply the rules of operations, or BODMAS to fraction calculations. Evaluate (a) $\frac{2}{3}\,\boldsymbol{of}\,3\frac{1}{2}+\frac{4}{5}$ (b) $\frac{2}{7}\left(1\frac{3}{4}+\frac{3}{8}\right)$ (c) $\frac{4}{9}+\frac{3}{4}\,\boldsymbol{of}\,2\frac{1}{5}$			
• I can solve problems involving fraction calculations. (1) A rectangle has length $3\frac{5}{7}$ cm and breadth $1\frac{2}{5}$ cm. Calculate its perimeter. (2) A triangle has base $2\frac{3}{4}$ cm and height $3\frac{2}{5}$ cm. Calculate its area. (2) Jamie is going to bake cakes for a party. He needs $\frac{2}{5}$ of a block of butter for 1 cake. He has 7 blocks of butter. How many cakes can Jamie bake?			

Learning Intention	I can compare two data sets using statistics.			

Success Criteria	☺	☺	☹
• I know that a 5 figure summary consists of the Lowest (L), Highest (H), median (Q2), lower quartile (Q1) and upper quartile (Q3) values in an **ordered data set**. The median (Q2) is the middle value. The lower quartile (Q1) is in the middle of the lower half and the upper quartile (Q3) is in the middle of the upper half of the ordered list.			
• I know how to construct a boxplot using a 5 figure summary.			
• I can make a 5 figure summary from a data set and draw a boxplot to illustrate the results. The marks obtained in a test were: 24 16 17 15 17 18 19 12 25 26 18 13 15 21 20 24 Find the maximum, minimum, median and quartiles of the data set and draw a boxplot to illustrate your results.			

• I know that the interquartile range and semi-interquartile range is a measure of spread of data.
• I can calculate the interquartile range (IQR) and semi-interquartile range (SIQR) from a data set using the
formulae $IQR = Q_3 - Q_1$ and $SIQR = \dfrac{Q_3 - Q_1}{2}$
 	
Before training athletes were tested on how many sit-ups they could do in one minute.
The following information was obtained :
lower quartile 23   median 39   upper quartile 51
 	
(a) Calculate the semi-interquartile range.
    After training the athletes were tested again.
    Both sets of data are displayed as boxplots.
 	
(b) Make **two** set of valid statements to compare the performances before and after training.			

Performance before training

Performance after training

number of sit-ups

Learning Intention	I can compare two data sets using statistics.			

Success Criteria	☺	😐	☹
• I can calculate the mean, $\bar{x}$ from a set of data using the formula $\bar{x} = \dfrac{\sum x}{n}$.			
• I know that standard deviation is a measure of spread of data.			
• I can calculate the standard deviation of a data set using the formula $s = \sqrt{\dfrac{\sum(x-\bar{x})^2}{n-1}}$ or $s = \sqrt{\dfrac{\sqrt{\sum x^2} - \dfrac{(\sum x)^2}{n}}{n-1}}$. A hotel inspector recorded the volume of wine, in millimetres, in a sample of six glasses. The results were 120 126 125 131 130 124 Use an appropriate formula to calculate the standard deviation.			
• I know that a high standard deviation, or SIQR, indicates data that is widely spread out from its mean. The terms more varied or less consistent describe the result			
• I know that a low standard deviation, or SIQR, indicates data is closer to the mean. The terms less varied or more consistent to describe the result.			
• I can make appropriate comments by comparing the means and standard deviations of two data sets. A group of people attended a course to help them stop smoking. The following table shows the statistics before and after the course. Make **two valid** comments about these results.			

	Mean number of cigarettes smoked per person per day	Standard Deviation
Before	20·8	8·5
After	9·6	12·0

Learning Intention	I can determine and use the equation of the line of best fit on a scatter graph.			

Success Criteria	☺	😐	☹
• I know that on a scattergraph we describe the relationship between the two quantities plotted as a **correlation**.			
• I can identify if there is a positive, negative or no correlation between two quantities. Positive Correlation Negative Correlation No Correlation			
• I can draw a line of best fit on a scatter graph. I know that approximately the same number of points should lie on each side of the line, the line should pass through at least two points and be extended to pass through the y-axis.			
• I can find the equation of the line of best fit using $y = mx + c$ or $y - b = m(x - a)$.			
• I can use the line of best fit to estimate one value given the other. The graph shows the relationship between the number of hours (h) a swimmer trains per week and the number of races (R) they have won. A best fitting straight line has been drawn. (a) Use information from the graph to find the equation of this line of best fit. (b) Use the equation to predict how many races a swimmer who trains 22 hours per week should win.			